Gravitation

Question1

The acceleration due to gravity on the surface of earth is g. If the diameter of earth reduces to half of its original value and mass remains constant, then acceleration due to gravity on the surface of earth would be :

[27-Jan-2024 Shift 1]

Options:

A.

g/4

B.

2g

C.

g/2

D.

4g

Answer: D

Solution:

$$g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$$

$$\frac{g_2}{g_1} = \frac{R_1^{\ 2}}{R_2^{\ 2}}$$

$$g_2 = 4g_1 \left(R_2 = \frac{R_1}{2} \right)$$

Question2

At what distance above and below the surface of the earth a body will have same weight, (take radius of earth as R.)

[29-Jan-2024 Shift 1]

Options:

A.

√5R − R

B.

$$\frac{\sqrt{3}R-R}{2}$$

C.

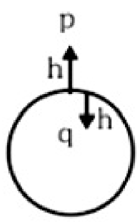
R/2

D.

$$\frac{\sqrt{5}R-R}{2}$$

Answer: D

Solution:



$$g_p = \frac{g^2}{(R+h)^2}$$

$$g_q = g\left(1 - \frac{h}{R}\right)$$

$$g_p = g_q$$

$$\frac{g}{\left(1+\frac{h}{R}\right)^2} = g\left(1 - \frac{h}{R}\right)$$

$$\left(1 - \frac{h^2}{R^2}\right)\left(1 + \frac{h}{R}\right) = 1$$

$$\text{Take} \quad \frac{h}{R} = x$$

So

$$x^3 - x + x^2 = 0$$

$$x = \frac{\sqrt{5}-1}{2}$$

$$h = \frac{R}{2}(\sqrt{5}-1)$$

Question3

A planet takes 200 days to complete one revolution around the Sun. If the distance of the planet from Sun is reduced to one fourth of the original distance, how many days will it take to complete one revolution?

[29-Jan-2024 Shift 2]

Options:

A.

25

B.

50

C.

100

D.

20

Answer: A

Solution:

$$T^2 \propto r^3$$

$$\frac{T_1^{\ 2}}{r_1^{\ 3}} = \frac{T_2^{\ 2}}{r_2^{\ 3}}$$

$$\frac{(200)^2}{r^3} = \frac{T_2^{\ 2}}{\left(\frac{r}{4}\right)^3}$$

$$\frac{200 \times 200}{4 \times 4 \times 4} = T_2^{\ 2}$$

$$T_2 = \frac{200}{4 \times 2}$$

$$T_2 = 25 \text{ days}$$

Question4

The gravitational potential at a point above the surface of earth is -5.12×10^7J/kg and the acceleration due to gravity at that point is 6.4m/ s^2. Assume that the mean radius of earth to be 6400km. The height of this point above the earth's surface is :

[30-Jan-2024 Shift 1]

Options:

A.

1600 km

B.

540 km

C.

1200 km

D.

1000 km

Answer: A

Solution:

$$-\frac{GM_E}{R_E+h} = -5.12 \times 10^{-7} \text{ ....(i)}$$

$$\frac{GM_E}{(R_E+h)^2} = 6.4 \text{........ (ii)}$$

By (i) and (ii)

$$\Rightarrow h = 16 \times 10^5 \text{m} = 1600 \text{ km}$$

Question5

Escape velocity of a body from earth is 11.2km/ s. If the radius of a planet be one-third the radius of earth and mass be one-sixth that of earth, the escape velocity from the plate is:

[30-Jan-2024 Shift 2]

Options:

A.

11.2km/ s

B.

8.4km/ s

C.

4.2km/ s

D.

7.9km/ s

Answer: D

Solution:

$R_P = \frac{R_E}{3}, M_P = \frac{M_E}{6}$

$V_e = \sqrt{\frac{2GM_e}{R_e}}$ ....(i)

$V_P = \sqrt{\frac{2GM_P}{R_P}}$ ....(ii)

$\frac{V_e}{V_p} = \sqrt{2}$

$V_P = \frac{V_e}{\sqrt{2}} = \frac{11.2}{\sqrt{2}} = 7.9\ \text{km}/\ sec$

Question6

Four identical particles of mass m are kept at the four corners of a square. If the gravitational force exerted on one of the masses by the other masses is $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$, the length of the sides of the square is

[31-Jan-2024 Shift 1]

Options:

A.

L/2

B.

4L

C.

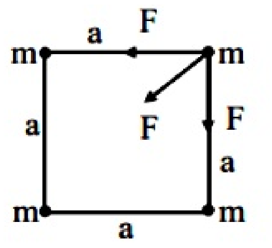
3L

D.

2L

Answer: B

Solution:



$$F_{net} = \sqrt{2}F + F'$$

$$F = \frac{Gm^2}{a^2} \text{ and } F' = \frac{Gm^2}{(\sqrt{2}a)^2}$$

$$F_{net} = \sqrt{2}\frac{Gm^2}{a^2} + \frac{Gm^2}{2a^2}$$

$$\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2} = \frac{Gm^2}{a^2}\left(\frac{2\sqrt{2}+1}{2}\right)$$

$$a = 4L$$

Question7

The mass of the moon is 1/144 times the mass of a planet and its diameter 1/16 times the diameter of a planet. If the escape velocity on the planet is v, the escape velocity on the moon will be:

[31-Jan-2024 Shift 2]

Options:

A.

V/3

B.

V/4

C.

C.

V/12

D.

V/6

Answer: A

Solution:

$$V_{escape} = \sqrt{\frac{2GM}{R}}$$

$$V_{planet} = \sqrt{\frac{2GM}{R}} = V$$

$$V_{Moon} = \sqrt{\frac{2GM \times 16}{144R}} = \frac{1}{3}\sqrt{\frac{2GM}{R}}$$

$$V_{Moon} = \frac{V_{Planet}}{3} = \frac{V}{3}$$

Question8

If R is the radius of the earth and the acceleration due to gravity on the surface of earth is $g = \pi^2 m/s^2$, then the length of the second's pendulum at a height h = 2R from the surface of earth will be,:

[1-Feb-2024 Shift 1]

Options:

A.

$\frac{2}{9}m$

B.

$\frac{1}{9}m$

C.

$\frac{4}{9}m$

D.

$\frac{8}{9}m$

Answer: B

Solution:

$$g' = \frac{GMe}{(3R)^2} = \frac{1}{9}g$$

$$T = 2\pi\sqrt{\frac{\ell}{g'}}$$

Since the time period of second pendulum is $2\ sec$.

$$T = 2\ sec$$

$$2 = 2\pi\sqrt{\frac{\ell}{g}9}$$

$$\ell = \frac{1}{9}m$$

Question9

A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T. If the force of attraction between planet and star is proportional to R $^{-3/2}$ then choose the correct option :

[1-Feb-2024 Shift 2]

Options:

A.

$T^2 \propto R^{5/2}$

B.

$T^2 \propto R^{7/2}$

C.

$T^2 \propto R^{3/2}$

D.

$T^2 \propto R^3$

Answer: A

Solution:

$$F = \frac{GMm}{R^{3/2}} = m\omega^2 R$$

$$\omega^2 \propto \frac{1}{R^{5/2}} \quad \because T = \frac{2\pi}{\omega} \quad \text{so}$$

$$T^2 \propto R^{5/2}$$

Question10

Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R .
Assertion A: A pendulum clock when taken to Mount Everest becomes fast.
Reason R: The value of g (acceleration due to gravity) is less at MountEverest than its value on the surface of earth.
In the light of the above statements, choose the most appropriate answer from the options given below
[24-Jan-2023 Shift 2]

Options:

A. Both A and R are correct but R is NOT the correct explanation of A

B. Both A and R are correct and R is the correct explanation of A

C. A is not correct but R is correct

D. A is correct but R is not correct

Answer: C

Solution:

Solution:
$T \propto \frac{1}{\sqrt{g}}$

Question11

The weight of a body at the surface of earth is 18 N. The weight of the body at an altitude of 3200 km above the earth's surface is (given, radius of earth $R_e = 6400\,km$)
[24-Jan-2023 Shift 1]

Options:

A. 9.8N

B. 4.9N

C. 19.6N

D. 8N

Answer: D

Solution:

Solution:
Acceleration due to gravity at height h

$$g' = \frac{g}{\left[1+\frac{h}{R}\right]^2}$$

So weight at given height

$$mg' = \frac{mg}{\left[1+\frac{h}{R}\right]^2} = \frac{18}{\left[1+\frac{1}{2}\right]^2} = 8N$$

Question12

Given below are two statements:
Statement I: Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface.
Statement II: Acceleration due to earth's gravity is same at a height ' h ' and depth ' d ' from earth's surface, if h = d.
In the light of above statements, choose the most appropriate answer form the options given below
[24-Jan-2023 Shift 2]

Options:

A. Statement I is incorrect but statement II is correct

B. Both Statement I and Statement II are incorrect

C. Statement I is correct but statement II is incorrect

D. Both Statement I and II are correct

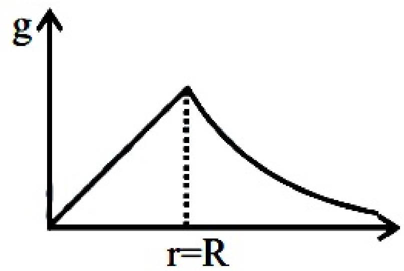
Answer: C

Solution:

Solution:

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$g' = g\left\{1-\frac{d}{R}\right\}$$



Statement I is correct & Statement II is incorrect

Question13

If the distance of the earth from Sun is 1.5×10^6 km. Then the distance of an imaginary planet from Sun, if its period of revolution is 2.83 years is:

[24-Jan-2023 Shift 2]

Options:

A. 6×10^7 km

B. 6×10^6 km

C. 3×10^6 km

D. 3×10^7 km

Answer: C

Solution:

Solution:

$T^2 \propto R^3 \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$

$\Rightarrow \left(\frac{1}{2.83}\right)^2 = \left(\frac{1.5 \times 10^6}{R_2}\right)^3$

$\Rightarrow R_2 = [(2.83)^2 \times (1.5 \times 10^6)^3]^{1/3}$

$= 8^{1/3} \times 1.5 \times 10^6 = 3 \times 10^6$ km

Question14

Assume that the earth is a solid sphere of uniform density and a tunnel is dug along its diameter throughout the earth. It is found that when a particle is released in this tunnel, it executes a simple harmonic motion. The mass of the particle is 100g. The time period of the motion of the particle will be (approximately)
(take $g = 10 ms^{-2}$, radius of earth $= 6400$ km)
[25-Jan-2023 Shift 1]

Options:

A. 24 hours

B. 1 hour 24 minutes

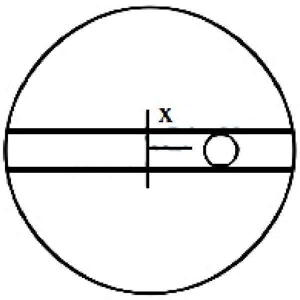
C. 1 hour 40 minutes

D. 12 hours

Answer: B

Solution:

Solution:



Let at some time particle is at a distance x from centre of Earth, then at that position field

$E = \frac{GM}{R^3}x$

∴ Acceleration of particle

$\vec{a} = -\frac{GM}{R^3}\vec{x}$

$\Rightarrow \omega = \sqrt{\frac{GM}{R^3}} = \sqrt{\frac{g}{R}}$

Now $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$

$\Rightarrow T = 2 \times 3.14 \times \sqrt{\frac{6400 \times 10^3}{10}}$

$= 2 \times 3.14 \times 800$ sec ≈ 1 hour 24 minutes

Question15

T is the time period of simple pendulum on the earth's surface. Its time period becomes x T when taken to a height R (equal to earth's radius) above the earth's surface. Then, the value of x will be: [25-Jan-2023 Shift 1]

Options:

A. 4

B. 2

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: B

Solution:

Solution:

At surface of earth time period

$T = 2\pi\sqrt{\frac{\ell}{g}}$

At height $h = R$

$g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2} = \frac{g}{4}$

$xT = 2\pi\sqrt{\frac{\ell}{(g/4)}}$

$\Rightarrow xT = 2 \times 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow xT = 2T \Rightarrow x = 2$

Question16

A body of mass is taken from earth surface to the height h equal to twice the radius of earth (R_e), the increase in potential energy will be : (g = acceleration due to gravity on the surface of Earth)
[25-Jan-2023 Shift 2]

Options:

A. 3 mgRe

B. $\frac{1}{3}mg_e$

C. $\frac{2}{3}mgR_e$

D. $\frac{1}{2}mgR_e$

Answer: C

Solution:

$U = \frac{-GM_e m}{r}$

$U_i = \frac{-GM_e m}{R_e}$

$U_f = \frac{-GM_e m}{(R_e + h)} = \frac{-GM_e m}{R_e + 2R_e}$

$\frac{-GM_e m}{3R_e}$

Increase in internal energy $\Delta U = U_f - U_i$

$= \frac{2}{3}\frac{GM_e m}{R_e}$

$\frac{2}{3}\frac{GM_e}{R_e{}^2}mR_e$

$= \frac{2}{3}mgR_e$

Question17

Every planet revolves around the sun in an elliptical orbit :
A. The force acting on a planet is inversely proportional to square of distance from sun.
B. Force acting on planet is inversely proportional to product of the masses of the planet and the sun
C. The centripetal force acting on the planet is directed away from the sun.
D. The square of time period of revolution of planet around sun is directly proportional to cube of semi-major axis of elliptical orbit.
Choose the correct answer from the options given below :

Options :
[25-Jan-2023 Shift 2]

Options:

A. A and D only

B. C and D only

C. B and C only

D. A and C only

Answer: A

Solution:

Solution:

$F = \frac{Gm_1m_2}{r^2}$

$\Rightarrow F \propto \frac{1}{r^2}$

$\Rightarrow F \propto m_1m_2$

$\Rightarrow$ This force provides centripetal force and acts towards sun

$\Rightarrow T^2 \propto a^3$ (Kepler's third law)

Question18

Two particles of equal mass ' m ' move in a circle of radius ' r ' under the action of their mutual gravitational attraction. The speed of each particle will be :
[29-Jan-2023 Shift 1]

Options:

A. $\sqrt{\frac{GM}{2r}}$

B. $\sqrt{\frac{4GM}{r}}$

C. $\sqrt{\frac{GM}{r}}$

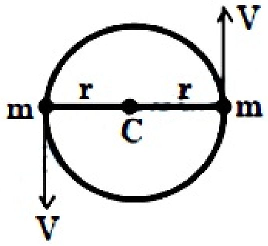
D. $\sqrt{\frac{GM}{4r}}$

Answer: D

Solution:

Solution:

$\frac{Gm^2}{4r^2} = \frac{mv^2}{r}$



$v = \sqrt{\frac{Cm}{4r}}$

Question19

The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become.
[29-Jan-2023 Shift 2]

Options:

A. 4 hours

B. 6 hours

C. 12 hours

D. 3 hours

Answer: D

Solution:

Solution:
Sol. $T^2 \propto R^3$

$$\frac{T_1^{\ 2}}{T_2^{\ 2}} = \frac{R_1^{\ 3}}{R_2^{\ 3}} \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R}{\frac{R}{4}}\right)^3$$

$$\therefore \ \frac{T_1^{\ 2}}{T_2^{\ 2}} = 64$$

$$\therefore \ T_2^{\ 2} = \frac{T_1^{\ 2}}{64}$$

$$\therefore \ T_2 = \frac{24}{8} = 3$$

Question20

If the gravitational field in the space is given as $\left(-\frac{K}{r^2}\right)$. Taking the reference point to be at $r = 2\,cm$ with gravitational potential $V = 10J / kg$. Find the gravitational potential at $r = 3\,cm$ in SI unit (Given, that $K = 6J\,cm / kg$)
[30-Jan-2023 Shift 1]

Options:

A. 9

B. 11

C. 12

D. 10

Answer: B

Solution:

Solution:

$$-\frac{dV}{dr} = -\frac{k}{r^2} \Rightarrow \int_{10}^{V} dV = \int_{2}^{3} \frac{k}{r^2} dr$$

$$V - 10 = k\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$V - 10 = \frac{k}{6} \Rightarrow V = 11 \text{ volts}$$

Question21

At a certain depth "d" below surface of earth. value of acceleration due to gravity becomes four times that of its value at a height 3R above earth surface. Where R is Radius of earth (Take R = 6400 km). The depth d is equal to
[31-Jan-2023 Shift 1]

Options:

A. 5260 km

B. 640 km

C. 2560 km

D. 4800 km

Answer: D

Solution:

Solution:

$$\frac{GM}{R^2}\left[1 - \frac{d}{R}\right] = \frac{4 \times GM}{(4R)^2}$$

$$1 - \frac{d}{R} = \frac{1}{4} \Rightarrow \frac{d}{R} = \frac{3}{4} \Rightarrow d \frac{3}{4}R$$

$$d = 4800 \text{ km}$$

Question22

Spherical insulating ball and a spherical metallic ball of same size and

mass are dropped from the same height. Choose the correct statement out of the following \{Assume negligible air friction\}
[31-Jan-2023 Shift 1]

Options:

A. Time taken by them to reach the earth's surface will be independent of the properties of their materials

B. Insulating ball will reach the earth's surface earlier than the metal ball

C. Both will reach the earth's surface simultaneously

D. Metal ball will reach the earth's surface earlier than the insulating ball.

Answer: B

Solution:

Solution:
When metal is passing through magnetic field, eddy current will produce and it will oppose the motion, so it will take more time.

Question23

A body weight W, is projected vertically upwards from earth's surface to reach a height above the earth which is equal to nine times the radius of earth. The weight of the body at that height will be:
[31-Jan-2023 Shift 2]

Options:

A. $\frac{W}{91}$

B. $\frac{W}{3}$

C. $\frac{W}{100}$

D. $\frac{W}{9}$

Answer: C

Solution:

Solution:

$$g_h = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

$$h = 9R$$

$$g_h = \frac{g}{(1+9)^2} = \frac{g}{100}$$

$$w_h = \frac{mg}{100} = \frac{w}{100}$$

Question24

Given below are two statements :
Statement-I: Acceleration due to gravity is different at different places on the surface of earth.
Statement-II: Acceleration due to gravity increases as we go down below the earth's surface. In the light of the above statements, choose the correct answer from the options given below
[1-Feb-2023 Shift 1]

Options:

A. Both Statement I and Statement II are true

B. Both Statement I and Statement II are false

C. Statement I is true but Statement II is false

D. Statement I is false but Statement II is true

Answer: C

Solution:

Solution:
$g_{eff} = g - \omega^2 R_e \sin^2\theta$, $\theta \rightarrow$ co-latitude angle
$g_{eff} = g\left(1 - \frac{d}{R_e}\right)$, d here depth

Question25

If earth has a mass nine times and radius twice to the of a planet P. Then $\frac{v_e}{3}\sqrt{x}\,ms^{-1}$ will be the minimum velocity required by a rocket to pull out of gravitational force of P, where v_e is escape velocity on earth. The value of x is
[1-Feb-2023 Shift 1]

Options:

A. 2

B. 3

C. 18

D. 1

Answer: A

Solution:

$$v_{(escape)\ plant} = \sqrt{\frac{2GM_P}{R_P}}$$

$$= \sqrt{\frac{2G\left(\frac{M_e}{9}\right)}{\left(\frac{R_e}{2}\right)}} = \frac{v_e\sqrt{2}}{3} \therefore x = 2$$

Question26

For a body projected at an angle with the horizontal from the ground, choose the correct statement.
[1-Feb-2023 Shift 2]

Options:

A. Gravitational potential energy is maximum at the highest point.

B. The horizontal component of velocity is zero at highest point.

C. The vertical component of momentum is maximum at the highest point.

D. The kinetic energy (K.E.) is zero at the highest point of projectile motion.

Answer: A

Solution:

Solution:
At highest point
$V_y = 0$
$V_x = u_x = u\cos\theta$
$U_g = mgh$, it is maximum at H_{max}.

Question27

The escape velocities of two planets A and B are in the ratio 1 : 2. If the ratio of their radii respectively is 1 : 3, then the ratio of acceleration due to gravity of planet A to the acceleration of gravity of planet B will be:
[1-Feb-2023 Shift 2]

Options:

A. $\frac{4}{3}$

B. $\frac{3}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

Answer: C

Solution:

Solution:

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\rho\frac{4}{3}\pi R^3}{R}} = C\sqrt{\rho}\cdot R$$

$$\frac{V_{e_1}}{V_{e_2}} = \frac{R_1}{R_2}\sqrt{\frac{\rho_1}{\rho_2}} = \frac{1}{2}$$

$$\frac{R_1{}^2}{R_2{}^2}\times\frac{\rho_1}{\rho_2} = \frac{1}{4}$$

$$\frac{R_1}{R_2} = \frac{1}{3}$$

$$g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3\times\rho}{R^2} C\cdot\rho R$$

$$\frac{g_1}{g_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{4}\frac{R_2{}^2}{R_1{}^2}\times\frac{R_1}{R_2}$$

$$= \frac{1}{4}\times\frac{R_2}{R_1} = \frac{3}{4}$$

Question28

A planet has double the mass of the earth. Its average density is equal to that of the earth. An object weighing W on earth will weigh on that planet:
[6-Apr-2023 shift 1]

Options:

A. $2^{1/3}W$

B. 2W

C. W

D. $2^{2/3}W$

Answer: A

Solution:

Solution:
Average Density of planet = average density of earth

$$\frac{M_e}{\frac{4}{3}\pi R_e{}^3} = \frac{M_p}{\frac{4}{3}\pi R_p{}^3}$$

$$\Rightarrow \frac{M_e}{R_e{}^3} = \frac{2M_e}{R_p{}^3}$$

$$\Rightarrow R_p = 2^{\frac{1}{3}}R_e ------- (i)$$

$$\text{Now,}\ \ g = \frac{GM}{R^2}$$

$$\frac{g_e}{g_p} = \frac{M_e}{R_e^2} \times \frac{R_p^2}{2M_e} = 2^{\frac{2}{3}-1} = 2^{-\frac{1}{3}}$$

$$\Rightarrow g_p = 2^{\frac{1}{3}} g_e$$

$$\Rightarrow W_p = 2^{\frac{1}{3}} W_e$$

Question29

Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A : Earth has atmosphere whereas moon doesn't have any atmosphere.
Reason R : The escape velocity on moon is very small as compared to that on earth.
In the light of the above statements. choose the correct answer from the options given below:
[6-Apr-2023 shift 1]

Options:

A. Both A and R are correct and R is the correct explanation of A

B. A is false but R is true

C. Both A and R are correct but R is NOT the correct explanation of A

D. A is true but R is false

Answer: A

Solution:

Solution:
$V_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{2gr}$
Radius of moon is less than that of earth and acceleration due to gravity is also less on moon as compared to that on earth.
Thus, V_{esc} of Moon $< V_{esc}$ of Earth
This is also the reason behind escape of atmosphere from moon.

Question30

The weight of a body on the surface of the earth is 100N. The gravitational force on it when taken at a height, from the surface of earth, equal to one-fourth the radius of the earth is :
[6-Apr-2023 shift 2]

Options:

A. 64N

B. 25N

C. 100N

D. 50N

Answer: A

Solution:

Solution:

using newton's formula $F = \frac{GM\,m}{r^2}$

at surface of earth, $100 = \frac{GM_e m}{Re^2}$ . . . (1)

at $r = R_e + \frac{R_e}{4} = \frac{5}{4}R_e$

$F' = \frac{GM_e m}{\left(\frac{5}{4}R_e\right)^2} = \frac{16}{25} \times \frac{GM_e m}{R_e{}^2}$

$F' = \frac{16}{25} \times 100 = 64N$

Question31

Choose the incorrect statement from the following : [6-Apr-2023 shift 2]

Options:

A. The linear speed of a planet revolving around the sun remains constant.

B. The speed of satellite in a given circular orbit remains constant.

C. When a body falls towards earth, the displacement of earth towards the body is negligible.

D. For a planet revolving around the sun in an elliptical orbit, the total energy of the planet remains constant.

Answer: A

Solution:

Solution:
Since planets revolve around the sun in an elliptical orbit its linear speed is not constant, hence option 1 not correct (and right choice).
Other statement are correct as per theory.

Question32

Given below are two statements:
Statement I : If E be the total energy of a satellite moving around the earth, then its potential energy will be $\frac{E}{2}$

Statement II : The kinetic energy of a satellite revolving in an orbit is equal to the half the magnitude of total energy E.
In the light of the above statements, choose the most appropriate answer from the options given below
[8-Apr-2023 shift 1]

Options:

A. Both Statement I and Statement II are incorrect

B. Statement I is incorrect but Statement II is correct

C. Statement I is correct but Statement II is incorrect

D. Both Statement I and Statement II are correct

Answer: A

Solution:

Solution:

For satellite K.E. $= \frac{1}{2}mv^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2$

$K.E. = \frac{GMm}{2r}$

Potential energy $U = -\frac{GMm}{r}$

Total energy $= K.E + U$

$E = -\frac{GMm}{2r}$

$U = 2E$ St I $-$ incorrect

K.E. $= |E|$ St II - incorrect

Question33

The weight of a body on the earth is 400N. Then weight of the body when taken to a depth half of the radius of the earth will be:
[8-Apr-2023 shift 1]

Options:

A. 300N

B. Zero

C. 100N

D. 200N

Answer: D

Solution:

Solution:
Weight on the earth surface $= mg$
$mg = 400N$ (given)

Weight at a depth $dw = m\left(\frac{GM(R-d)}{R^3}\right)$

$W = mg\left(1 - \frac{d}{R}\right)$

$d = \frac{R}{2} \Rightarrow w = mg\left(1 - \frac{1}{2}\right) \Rightarrow w = \frac{mg}{2}$

$w = 200N$

Question34

The orbital angular momentum of a satellite is L, when it is revolving in a circular orbit at height h from earth surface. If the distance of satellite from the earth center is increased by eight times to its initial value, then the new angular momentum will be-
[8-Apr-2023 shift 2]

Options:

A. 8L

B. 3L

C. 4L

D. 9L

Answer: B

Solution:

Solution:

$L = mvv_0r\left(v_0 = \sqrt{\frac{GM}{h}}\right)$

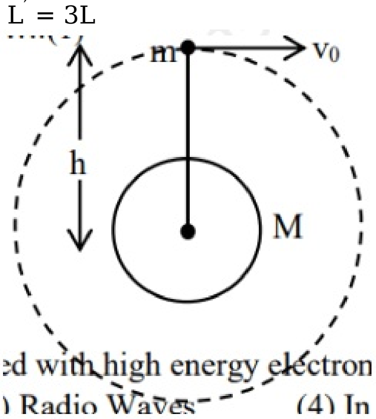
$L = m\sqrt{GMh}$

$h^{'} \rightarrow h + 8h = 9h \ldots (1)$

$L^{'} = m\sqrt{GM\,9\,h} \ldots (2)$

$\frac{L^{'}}{L} = 3$

$L^{'} = 3L$



Question35

The acceleration due to gravity at height h above the earth if h< < R

(Radius of earth) is given by
[8-Apr-2023 shift 2]

Options:

A. $g' = g\left(1 - \frac{h^2}{2R^2}\right)$

B. $g' = g\left(1 - \frac{h}{2R}\right)$

C. $g' = g\left(1 - \frac{2h^2}{R^2}\right)$

D. $g' = g\left(1 - \frac{2h}{R}\right)$

Answer: D

Solution:

Solution:

$g' = \frac{GM}{(R+h)^2}$

$g' = \frac{GM}{R^2\left(1 + \frac{h}{R}\right)^2}$

using binomial expansion & neglect higher order term

$\Rightarrow g = g\left(1 - \frac{2h}{R}\right)$

Question36

Two satellites of masses m and 3m revolve around the earth in circular orbits of radii r&3r respectively. The ratio of orbital speeds of the satellites respectively is
[10-Apr-2023 shift 1]

Options:

A. 3 : 1

B. 1 : 1

C. $\sqrt{3} : 1$

D. 9 : 1

Answer: C

Solution:

Solution:

$v = \sqrt{\frac{GM}{r}} \Rightarrow v \times \frac{1}{\sqrt{r}}$; $M =$ mass of earth, $r =$ radius of earth

$$\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{3r}{r}} = \sqrt{3}$$

Question37

Assuming the earth to be a sphere of uniform mass density, the weight of a body at a depth d = $\frac{R}{2}$ from the surface of earth, if its weight on the surface of earth is 200N, will be :
[10-Apr-2023 shift 1]

Options:

A. 500N

B. 400N

C. 100N

D. 300N

Answer: C

Solution:

Solution:
$mg = 200N$

$$g' = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{R}{2 \times R}\right) = \frac{g}{2}$$

$$\text{weight} = mg' = \frac{mg}{2} = \frac{200}{2} = 100N$$

Question38

If the earth suddenly shrinks to $\frac{1}{64}$ th of its original volume with its mass remaining the same, the period of rotation of earth becomes $\frac{24}{x}$h. The value of x is _______.
[10-Apr-2023 shift 1]

Answer: 16

Solution:

Solution:
By AMC

$$\frac{2}{5}MR^2\omega_1{}^2 = \frac{2}{5}M\left(\frac{R}{4}\right)^2\omega_2$$

$$\frac{\omega_1}{\omega_2} = \frac{1}{16} = \frac{T_2}{T_1} = \frac{T_2}{24}$$
$T_2 = \frac{24}{16}$ $\therefore x = 16$ Ans.

Question39

Given below are two statements:
Statement I : Rotation of the earth shows effect on the value of acceleration due to gravity (g)
Statement II : The effect of rotation of the earth on the value of ' g ' at the equator is minimum and that at the pole is maximum.
In the light of the above statements, choose the correct answer from the options given below.
[10-Apr-2023 shift 2]

Options:

A. Both Statement I and Statement II are true

B. Both Statement I and Statement II are false

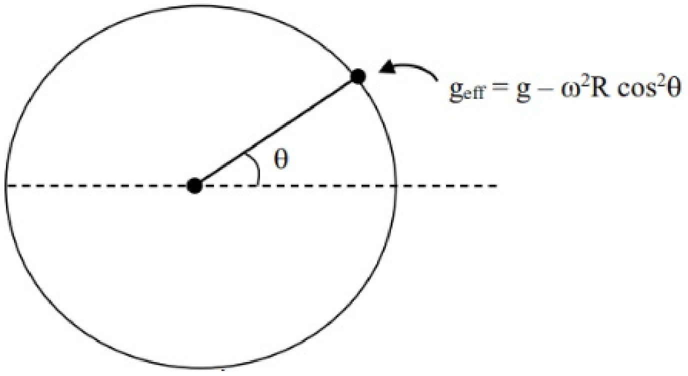
C. Statement I is false but statement II is true

D. Statement I is true but statement II is false

Answer: D

Solution:

Solution:
Due to rotation of earth, $g_{eff} = g - \omega^2 R\cos^2\theta$
Where ' θ ' is angle made with equator
Also, At poles, $\theta = 90^\circ$
$\Delta g = \omega^2 R\cos^2\theta$
$= \omega^2 R\cos^2 90 = 0$
[no effect on poles]
$g_{eff} = g - \omega^2 R\cos^2\theta$
for equator $\theta = 0^\circ$
So, $g_{eff} = g - w^2 R$
$\Delta g = \omega^2 R$ (Which is maximum change)



Question40

The time period of a satellite, revolving above earth's surface at a height equal to R will be (Given $g = \pi^2 m / s^2$, R = radius of earth)
[10-Apr-2023 shift 2]

Options:

A. $\sqrt{32R}$

B. $\sqrt{4R}$

C. $\sqrt{2R}$

D. $\sqrt{8R}$

Answer: A

Solution:

Solution:

$T^2 = \frac{4\pi^2 r^3}{GM}$

$T^2 = \frac{4\pi^2(2R)^3}{GM}$

$= 4 \times 8 \times \frac{\pi 2 \times R^3}{GM}$

$= 4 \times 8 \times \frac{(g)(R^3)}{GR^2}$

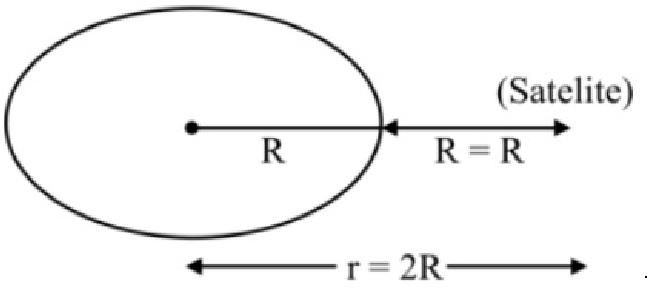
At surface of earth $\Rightarrow g = \frac{GM}{R^2}$

So, $GM = g \cdot R^2$

Also $\pi^2 = g$

$T^2 = 32R$

$\Rightarrow T = \sqrt{32R}$



Question41

The radii of two planets 'A' and 'B' are 'R' and '4R' and their densities are ρ and ρ / 3 respectively. The ratio of acceleration due to gravity at their surfaces ($g_A : g_B$) will be :
[11-Apr-2023 shift 1]

Options:

A. 1 : 16

B. 3 : 16

C. 3 : 4

D. 4 : 3

Answer: C

Solution:

Solution:

$g = \frac{4\pi}{3} GR\delta$

$g \propto \delta R$

$\frac{g_A}{g_B} = \frac{\delta_A R_A}{\delta_B \cdot R_B} = \frac{\delta \cdot R}{\frac{\delta}{3} \cdot 4R} = \frac{3}{4}$

Question42

A space ship of mass 2×10^4 kg is launched into a circular orbit close to the earth surface. The additional velocity to be imparted to the space ship in the orbit to overcome the gravitational pull will be (if $g = 10 m / s^2$ and radius of earth $= 6400$ km):
[11-Apr-2023 shift 2]

Options:

A. $7.9(\sqrt{2} - 1)$ km / s

B. $7.4(\sqrt{2} - 1)$ km / s

C. $11.2(\sqrt{2} - 1)$ km / s

D. $8(\sqrt{2} - 1)$ km / s

Answer: D

Solution:

Solution:

$\Delta V = \sqrt{\frac{GM}{R}}(\sqrt{2} - 1)$

$= \sqrt{\frac{GM}{R^2} \times R}(\sqrt{2} - 1)$

$= \sqrt{gR}(\sqrt{2} - 1) = 8000(\sqrt{2} - 1) ms^{-1}$

$= 8(\sqrt{2} - 1) kms^{-1}$

Question43

Two satellites A and B move round the earth in the same orbit. The mass of A is twice the mass of B. The quantity which is same for the two satellites will be
[12-Apr-2023 shift 1]

Options:

A. Kinetic energy

B. Speed

C. Total energy

D. Potential energy

Answer: B

Solution:

Solution:

Speed $V = \sqrt{\frac{GM_e}{r}}$

$m_e =$ mass of earth

$r =$ radius of orbit

Independent on mass of satellite so speed is same Other three quantities are mass dependent

$\rightarrow KE = \frac{Gm_e m}{2r}$

$\rightarrow PE = \frac{-Gm_e m}{r}$

$\rightarrow TE = \frac{-Gm_e m}{2r}$

Question44

The ratio of escape velocity of a planet to the escape velocity of earth will be :-
Given : Mass of the planet is 16 times mass of earth and radius of the planet is 4 times the radius of earth.
[12-Apr-2023 shift 1]

Options:

A. 1 : 4

B. 4 : 1

C. 2 : 1

D. $1 : \sqrt{2}$

Answer: C

Solution:

Solution:
Given mass of planet $= 16$ mass of the earth $(m_p = 16m_e)$

$r_p = 4r_o$

We know $V_{escape} = \sqrt{\frac{2Gm}{R}}, \frac{V_{esplanet}}{V_{esearth}} = ?$

$$\frac{V_{\text{es plavet}}}{V_{\text{es earth}}} = \sqrt{\frac{2GM_p}{R_p} \times \frac{R_e}{2GM_e}}$$
$$= \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$$
$$= \sqrt{\frac{16m_e}{m_e} \times \frac{R_e}{4R_e}}$$
$$= \sqrt{\frac{16}{4}} = 2 : 1$$
$$\frac{V_{\text{esplanet}}}{V_{\text{eseath}}} = \frac{2}{1}$$

Question45

A planet having mass 9 Me and radius 4 Re, where Me and Re are mass and radius of earth respectively, has escape velocity in km / s given by: (Given escape velocity on earth $V_e = 11.2 \times 10^3 m / s$)
[13-Apr-2023 shift 1]

Options:

A. 11.2

B. 67.2

C. 33.6

D. 16.8

Answer: D

Solution:

Solution:

Escape velocity, $v_e = \sqrt{\frac{2GM}{R}}$

$$V_p = \sqrt{\frac{2G(9m_e)}{4R_e}} = \frac{3}{2}(V_e)_{\text{earth}}$$
$$v_p = \frac{3}{2} \times 11.2\,\text{km / s}$$
$$v_p = 16.8\,\text{km / s}$$

Question46

Two planets A and B of radii R and 1.5R have densities ρ and ρ / 2 respectively. The ratio of acceleration due to gravity at the surface of B to A is :
[13-Apr-2023 shift 2]

Options:

A. 2 : 3

B. 2 : 1

C. 4 : 3

D. 3 : 4

Answer: D

Solution:

Solution:

$$g = \frac{GM}{R^2} = \frac{G \times \frac{4}{3}\pi R^3 \times \rho}{R^2} = \frac{4}{3}G\pi PR$$

$$\frac{g_2}{g_1} = \frac{R_2}{R_1} \times \frac{\rho_2}{\rho_1} = 1.5 \times \frac{1}{2} = \frac{3}{4}$$

Question47

Given below are two statements :
Statement I : For a planet, if the ratio of mass of the planet to its radius increases, the escape velocity from the planet also increases.
Statement II : Escape velocity is independent of the radius of the planet.
In the light of above statements, choose the most appropriate answer form the options given below
[13-Apr-2023 shift 2]

Options:

A. Both Statement I and Statement II are correct

B. Statement I is correct but statement II is incorrect

C. Statement I is incorrect but statement II is correct

D. Both Statement I and Statement II are incorrect

Answer: B

Solution:

Solution:

$$V_e = \sqrt{\frac{2GM}{R}}$$

As, $\frac{M}{R}$ increases $\Rightarrow V_e$ increases

$$V_e \propto \frac{1}{\sqrt{R}}$$

As, V_e depends on R

Question48

The approximate height from the surface of earth at which the weight of

the body becomes $\frac{1}{3}$ of its weight on the surface of earth is :
[Radius of earth R = 6400 km and $\sqrt{3}$ = 1.732]
[24-Jun-2022-Shift-1]

Options:

A. 3840 km

B. 4685 km

C. 2133 km

D. 4267 km

Answer: B

Solution:

Solution:

$M' = \frac{M}{3}g$

$g' = \frac{g}{3}$

$g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{3}$

$\frac{R}{R+h} = \frac{1}{\sqrt{3}}$

$h = (\sqrt{3} - 1)R$

$= (1.732 - 1)6400$

$h = 4685 \text{ km}$

Question49

The distance between Sun and Earth is R. The duration of year if the distance between Sun and Earth becomes 3R will be:
[24-Jun-2022-Shift-2]

Options:

A. $\sqrt{3}$ years

B. 3 years

C. 9 years

D. $3\sqrt{3}$ years

Answer: D

Solution:

Solution:

We know that$T^2 \propto R^3$

$\Rightarrow \left(\frac{T'}{T}\right)^2 = \left(\frac{3R}{R}\right)^3$

$\Rightarrow \frac{T'}{T} = 3\sqrt{3}$

$\Rightarrow T' = 3\sqrt{3}$ years

Question50

The height of any point P above the surface of earth is equal to diameter of earth. The value of acceleration due to gravity at point P will be: (Given g = acceleration due to gravity at the surface of earth). [25-Jun-2022-Shift-1]

Options:

A. g / 2

B. g / 4

C. g / 3

D. g / 9

Answer: D

Solution:

$h = 2R$

$\therefore g' = \dfrac{GM}{(R+h)^2}$

$= \dfrac{GM}{9R^2}$

$= \dfrac{g}{9}$

Question51

Two satellites S_1 and S_2 are revolving in circular orbits around a planet with radius $R_1 = 3200\,km$ and $R_2 = 800\,km$ respectively. The ratio of speed of satellite S_1 to be speed of satellite S_2 in their respective orbits would be $\frac{1}{x}$ where x =
[25-Jun-2022-Shift-2]

$$v = \sqrt{\frac{GM}{R}}$$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{R_2}{R_1}}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{3200}{800}} = 2$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{1}{2}$$

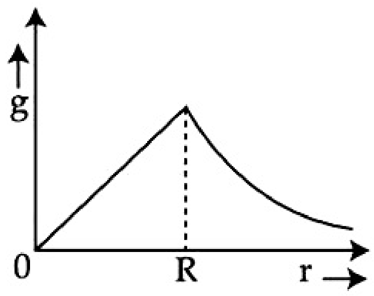
$x = 2$

Question52

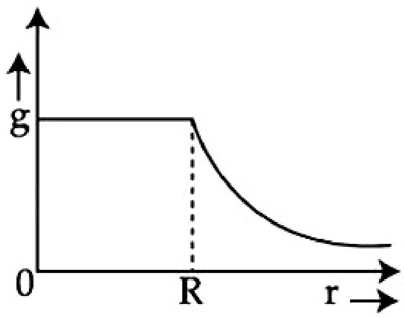
The variation of acceleration due to gravity (g) with distance (r) from the center of the earth is correctly represented by :
(Given R = radius of earth)
[26-Jun-2022-Shift-1]

Options:

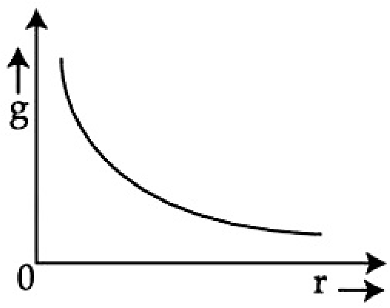
A.



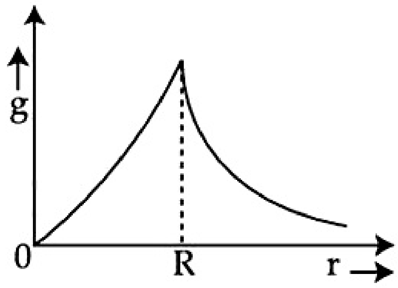
B.



C.



D.



Answer: A

Solution:

Solution:

For $r < R g = \frac{Gmr}{R^3} = Cr$ ($C =$ Constant)

For $r > R g = \frac{Gm}{r^2} = \frac{C'}{r^2}$ ($C' =$. Constant)

For the above equations the best suited graph is as given in option (A)

Question53

The elongation of a wire on the surface of the earth is 10^{-4}m. The same wire of same dimensions is elongated by 6×10^{-5}m on another planet. The acceleration due to gravity on the planet will be ___ ms^{-2}. (Take acceleration due to gravity on the surface of earth $= 10ms^{-2}$)
[26-Jun-2022-Shift-1]

Solution:

on earth, $\Delta l = 10^{-4}m$

on other planet $\Delta l' = 6 \times 10^{-5}m$

$$\Delta l = \frac{Fl}{Ay} \Rightarrow \frac{\Delta l'}{\Delta l} = \frac{\frac{F'l}{Ay}}{\frac{Fl}{Ay}} = \frac{mg'}{mg}$$

$$\Rightarrow g' = \frac{\Delta l'}{\Delta l} \times g$$

$$= \frac{6 \times 10^{-5}}{10^{-4}} \times 10$$

$$\Rightarrow g' = 6m / s^2$$

Question54

Given below are two statements : One is labelled as Assertion A and the

other is labelled as Reason R.
Assertion A : If we move from poles to equator, the direction of acceleration due to gravity of earth always points towards the center of earth without any variation in its magnitude.
Reason R: At equator, the direction of acceleration due to the gravity is towards the center of earth.
In the light of above statements, choose the correct answer from the options given below:
[26-Jun-2022-Shift-2]

Options:

A. Both A and R are true and R is the correct explanation of A.

B. Both A and R are true but R is NOT the correct explanation of A.

C. A is true but R is false.

D. A is false but R is true.

Answer: D

Solution:

Solution:
$g' = g_0 - \omega^2 R\cos^2\theta$
$\theta =$ latitude.

Question55

Two identical particles each of mass ' m ' go round a circle of radius a under the action of their mutual gravitational attraction. The angular speed of each particle will be :
[15-Apr-2023 shift 1]

Options:

A. $\sqrt{\frac{Gm}{a^3}}$

B. $\sqrt{\frac{Gm}{4a^3}}$

C. $\sqrt{\frac{Gm}{2a^3}}$

D. $\sqrt{\frac{Gm}{8a^3}}$

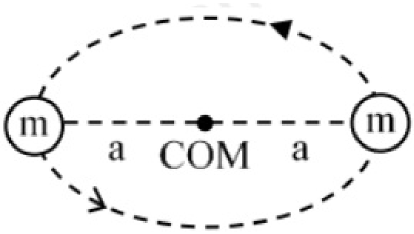
Answer: B

Solution:

$F_C = m\omega^2 a$

$\frac{Gm^2}{(2a)^2} = m\omega^2 a$

$\omega = \sqrt{\frac{Gm}{4a^3}}$



Question56

Given below are two statements :
Statement I: The law of gravitation holds good for any pair of bodies in the universe.
Statement II : The weight of any person becomes zero when the person is at the centre of the earth.
In the light of the above statements, choose the correct answer from the options given below.
[27-Jun-2022-Shift-1]

Options:

A. Both Statement I and Statement II are true

B. Both Statement I and Statement II are false

C. Statement I is true but Statement II is false

D. Statement I is false but Statement II is true

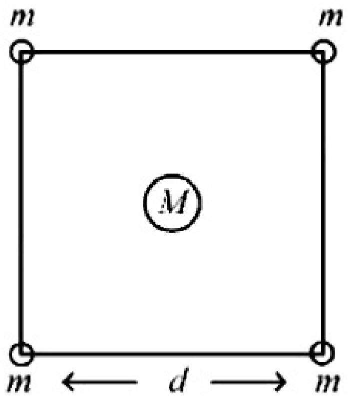
Answer: A

Solution:

Solution:
Statement - I is true as law of gravitation is a universal law.
Statement - II is also true as gravitational field at centre of earth is zero.

Question57

Four spheres each of mass m from a square of side d (as shown in figure). A fifth sphere of mass M is situated at the centre of square. The total gravitational potential energy of the system is :



[27-Jun-2022-Shift-2]

Options:

A. $-\frac{Gm}{d}[(4+\sqrt{2})m+4\sqrt{2}m]$

B. $-\frac{Gm}{d}[(4+\sqrt{2})M\ +4\sqrt{2}m]$

C. $-\frac{Gm}{d}[3m^2+4\sqrt{2}M\ ]$

D. $-\frac{Gm}{d}[6m^2+4\sqrt{2}M\ ]$

Answer: A

Solution:

Solution:
Total gravitational potential energy

$$= -\left\{\frac{4GM\,m}{d\ /\sqrt{2}}+\frac{4Gm^2}{d}+\frac{2Gm^2}{\sqrt{2}d}\right\}$$

$$= -\frac{Gm}{d}\{M\,4\sqrt{2}+(4+\sqrt{2})m\}$$

$$= -\frac{Gm}{d}\{4\sqrt{2}M\ +(4+\sqrt{2})m\}$$

Question58

Two planets A and B of equal mass are having their period of revolutions T_A and T_B such that $T_A = 2T_B$. These planets are revolving in the circular orbits of radii r_A and r_B respectively. Which out of the following would be the correct relationship of their orbits?
[28-Jun-2022-Shift-1]

Options:

A. $2r_A^{\ 2} = r_B^{\ 3}$

B. $r_A^{\ 3} = 2r_B^{\ 3}$

C. $r_A^{\ 3} = 4r_B^{\ 3}$

D. $T_A^2 - T_B^2 = \frac{\pi^2}{GM}(r_B^3 - 4r_A^3)$

Answer: C

Solution:

Solution:

$T_A = 2T_B$

Now $T_A^2 \propto r_A^3$

$\Rightarrow \left(\frac{r_A}{r_B}\right)^3 = \left(\frac{T_A}{T_B}\right)^2$

$\Rightarrow r_A^3 = 4r_B^3$

Question59

Water falls from a 40m high dam at the rate of 9×10^4 kg per hour. Fifty percentage of gravitational potential energy can be converted into electrical energy. Using this hydroelectric energy number of 100W lamps, that can be lit, is :
(. Take .g = $10ms^{-2}$)
[28-Jun-2022-Shift-2]

Options:

A. 25

B. 50

C. 100

D. 18

Answer: B

Solution:

Solution:

Total gravitational PE of water per second $= \frac{mgh}{T}$

$= \frac{9 \times 10^4 \times 10 \times 40}{3600} = 10^4 J / \text{ sec}$

50% of this energy can be converted into electrical energy so total electrical energy $= \frac{10^4}{2} = 5000$ W

So total bulbs lit can be $= \frac{5000W}{100W} = 50$ bulbs

Question60

Two objects of equal masses placed at certain distance from each other attracts each other with a force of F . If one-third mass of one object is transferred to the other object, then the new force will be :

[28-Jun-2022-Shift-2]

Options:

A. $\frac{2}{9}F$

B. $\frac{16}{9}F$

C. $\frac{8}{9}F$

D. F

Answer: C

Solution:

Solution:

Let the masses are m and distance between them is l, then $F = \frac{Gm^2}{l^2}$.

When $1/3^{rd}$ mass is transferred to the other then masses will be $\frac{4m}{3}$ and $\frac{2m}{3}$. so new force will be

$$F' = \frac{G\frac{4m}{3} \times \frac{2m}{3}}{l^2} = \frac{8}{9}\frac{Gm^2}{l^2} = \frac{8}{9}F$$

Question61

The escape velocity of a body on a planet ' A ' is $12kms^{-1}$. The escape velocity of the body on another planet ' B ', whose density is four times and radius is half of the planet ' A ', is :
[29-Jun-2022-Shift-1]

Options:

A. $12kms^{-1}$

B. $24kms^{-1}$

C. $36kms^{-1}$

D. $6kms^{-1}$

Answer: A

Solution:

Solution:

$$v_{esc} - \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R} \times \rho \times \frac{4}{3}\pi R^3}$$

$$\Rightarrow v_{esc} \propto R\sqrt{\rho}$$

$$\Rightarrow \frac{(v_{esc})_B}{(v_{esc})_A} = 1$$

$\Rightarrow (v_{esc})_B = 12\,km/s$

Question62

The time period of a satellite revolving around earth in a given orbit is 7 hours. If the radius of orbit is increased to three times its previous value, then approximate new time period of the satellite will be [29-Jun-2022-Shift-2]

Options:

A. 40 hours

B. 36 hours

C. 30 hours

D. 25 hours

Answer: B

Solution:

Solution:

$T_2^{\ 2} = \left(\frac{R_2}{R_1}\right)^3 T_1^{\ 2}$

$\Rightarrow T_2 = (3)^{3/2} \times 7 \approx 5.2 \times 7$

$T_2 \cong 36\,hrs$

Question63

Three identical particles A, B and C of mass 100kg each are placed in a straight line with AB = BC = 13m. The gravitational force on a fourth particle P of the same mass is F, when placed at a distance 13m from the particle B on the perpendicular bisector of the line AC. The value of F will be approximately:
[25-Jul-2022-Shift-1]

Options:

A. 21G

B. 100G

C. 59G

D. 42G

Answer: B

Solution:

$m = 100\text{kg}$

$F_{AP} = \frac{Gm^2}{(13\sqrt{2})^2}$

$F_{BP} = \frac{Gm^2}{13^2}$

$F_{CP} = \frac{Gm^2}{(13\sqrt{2})^2}$

$F_{net} = F_{BP} + F_{AP}\cos 45^\circ + F_{CP}\cos 45^\circ$

$= \frac{Gm^2}{13^2}\left(1 + \frac{1}{\sqrt{2}}\right)$

$= \frac{G100^2}{169}(1 + 0.707)$

$\simeq 100G$

Question64

An object is taken to a height above the surface of earth at a distance $\frac{5}{4}$R from the centre of the earth. Where radius of earth, R = 6400km. The percentage decrease in the weight of the object will be [25-Jul-2022-Shift-2]

Options:

A. 36%

B. 50%

C. 64%

D. 25%

Answer: A

Solution:

$w = mg$

$w' = \frac{mg}{\left(1 + \frac{h}{R}\right)^2} = \frac{mg}{\left(\frac{5}{4}\right)^2} = \frac{16}{25}mg$

$\therefore$% decrease in weight

$= \left(1 - \frac{16}{25}\right) \times 100\%$

$= 36\%$

Question65

The percentage decrease in the weight of a rocket, when taken to a height of 32 km above the surface of earth will, be :
(Radius of earth = 6400 km)
[26-Jul-2022-Shift-1]

Options:

A. 1%

B. 3%

C. 4%

D. 0.5%

Answer: A

Solution:

GM

$$\Rightarrow \frac{\Delta g}{g} = 2\frac{\Delta r}{r}$$

$$\Rightarrow \frac{\Delta g}{g} \times 100 = 2 \times \frac{32}{6400} \times 100\% = 1\%$$

$\Rightarrow$% decrease in weight = 1%

Question66

A body is projected vertically upwards from the surface of earth with a velocity equal to one third of escape velocity. The maximum height attained by the body will be :
(Take radius of earth = 6400 km and g = $10ms^{-2}$)
[26-Jul-2022-Shift-2]

Options:

A. 800 km

B. 1600 km

C. 2133 km

D. 4800 km

Answer: A

Solution:

Solution:

$$v_e = \sqrt{\frac{2Gm}{R}}$$

$$\frac{-GMm}{R} + \frac{1}{2}m\frac{V_e}{2}9 = -\frac{GMm}{R+h}$$
$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{V_e{}^2}{18}$$
$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{GM}{9R}$$
$$\frac{GM}{R+h} = \frac{8\,GM}{9R}$$
$$\frac{1}{R+h} = \frac{8}{9R}$$
$$9R = 8R + 8h$$
$$h = \frac{R}{8} \Rightarrow \frac{6400}{8} \Rightarrow 800\,\text{km}$$

Question67

Two satellites A and B, having masses in the ratio 4 : 3, are revolving in circular orbits of radii 3r and 4r respectively around the earth. The ratio of total mechanical energy of A to B is :
[27-Jul-2022-Shift-1]

Options:

A. 9 : 16

B. 16 : 9

C. 1 : 1

D. 4 : 3

Answer: B

Solution:

Solution:

Given that $\frac{m_1}{m_2} = \frac{4}{3}, \frac{r_1}{r_2} = \frac{3}{4}$

Now TE $= \frac{1}{2}mv^2 + \left(\frac{-GMm}{r}\right)$

but $\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow mv^2 = \frac{GMm}{r}$

$\Rightarrow TE = -\frac{GMm}{2r} \propto \frac{m}{r}$

$$\frac{TE_1}{TE_2} = \frac{m_1}{m_2}\cdot\frac{r_2}{r_1} = \frac{4}{3}\times\frac{4}{3} = \frac{16}{9}$$

Question68

A body of mass m is projected with velocity λv_e in vertically upward direction from the surface of the earth into space. It is given that v_e is escape velocity and $\lambda < 1$. If air resistance is considered to be negligible, then the maximum height from the centre of earth, to which the body can go, will be :

(R : radius of earth)
[27-Jul-2022-Shift-2]

Options:

A. $\frac{R}{1+\lambda^2}$

B. $\frac{R}{1-\lambda^2}$

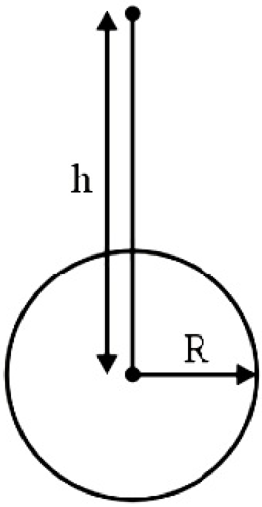
C. $\frac{R}{1-\lambda}$

D. $\frac{\lambda^2 R}{1-\lambda^2}$

Answer: B

Solution:

Solution:



$$-\frac{GMm}{R}+\frac{1}{2}m\lambda^2 V_e{}^2=-\frac{GMm}{h}$$
$$-\frac{GMm}{R}+\frac{1}{2}\lambda^2\frac{2\,GMm}{R}=-\frac{GMm}{h}$$
$$\frac{\lambda^2}{R}-\frac{1}{R}=\frac{-1}{h}$$
$$\frac{1}{h}=\frac{1-\lambda^2}{R}$$
$$h=\frac{R}{1-\lambda^2}$$

Question69

If the radius of earth shrinks by 2% while its mass remains same. The acceleration due to gravity on the earth's surface will approximately : [28-Jul-2022-Shift-1]

Options:

A. decrease by 2%

B. decrease by 4%

C. increase by 2%

D. increase by 4%

Answer: D

Solution:

Solution:

$g = \frac{GM}{R^2}$

$M =$ constant $g < \frac{1}{R^2}$

$100\frac{\Delta g}{g} = -2\frac{\Delta R}{R}100$

% change $= -2(-2)$

% change in g = 4%

increase by 4%

Question70

Assume there are two identical simple pendulum clocks. Clock −1 is placed on the earth and Clock - 2 is placed on a space station located at a height h above the earth surface. Clock - 1 and Clock - 2 operate at time periods 4 s and 6s respectively. Then the value of h is___ (consider radius of earth $R_E = 6400$ km and g on earth 10m / s^2) [28-Jul-2022-Shift-2]

Options:

A. 1200 km

B. 1600 km

C. 3200 km

D. 4800 km

Answer: C

Solution:

Solution:

$T \propto \sqrt{1/g}$

$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \frac{R}{R+h}$

$\frac{4}{6} = \frac{R}{R+h}$

$\Rightarrow h = R/2$

$= 3200$ km

Question71

If the acceleration due to gravity experienced by a point mass at a height h above the surface of earth is same as that of the acceleration due to gravity at a depth $\alpha h(h < < R_e)$ from the earth surface. The value of α will be ________.
(use $R_e = 6400\,km$)
[29-Jul-2022-Shift-1]

Solution:

Solution:
$g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$
$\frac{2h}{R} = \frac{d}{R}$
$\alpha h = d$
$\alpha = 2$

Question72

An object of mass 1 kg is taken to a height from the surface of earth which is equal to three times the radius of earth. The gain in potential energy of the object will be [If, $g = 10ms^{-2}$ and radius of earth $= 6400\,km$]
[29-Jul-2022-Shift-2]

Options:

A. 48 MJ

B. 24 MJ

C. 36 MJ

D. 12 MJ

Answer: A

Solution:

Solution:
$\Delta U = U_f - U_i$
$= -\frac{GMm}{4R} + \frac{GMm}{R}$
$= \frac{3GMm}{4R} = \frac{3}{4}mgR$
$= 48\,MJ$

Question73

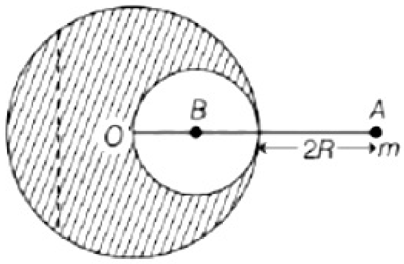
A cylindrical wire of radius 0.5mm and conductivity $5 \times 10^7 \mathrm{S} / \mathrm{m}$ is subjected to an electric field of $10 \mathrm{mV} / \mathrm{m}$. The expected value of current in the wire will be $x^3 \pi \mathrm{mA}$. The value of x is ....
[24 Feb 2021 Shift 2]

Solution:

Given, radius of cylindrical wire, $r = 0.5 \mathrm{mm} = 0.5 \times 10^{-3} \mathrm{m}$
Conductivity, $\sigma = 5 \times 10^7 \mathrm{S} / \mathrm{m}$
$E = 10 \mathrm{mV} / \mathrm{m} = 10 \times 10^{-3} \mathrm{V} / \mathrm{m}$
t current density,
$\therefore J = \sigma E$
$= 5 \times 10^7 \times 10 \times 10^{-3} = 5 \times 10^5 \mathrm{A} / \mathrm{m}^2$
Also, $J = I / A \Rightarrow I = JA$
$\Rightarrow I = 5 \times 10^5 \times \pi \times (0.5 \times 10^{-3})^2$
$= 5 \times 10^5 \times \pi \times 25 \times 10^{-8} = 125\pi \times 10^{-3}$
$\Rightarrow x^3 \pi \mathrm{mA} = 125 \pi \mathrm{mA} \Rightarrow x^3 = 5^3$
$\Rightarrow x = 5$

Question74

A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now, a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is



[25 Feb 2021 Shift 1]

Options:

A. 25 : 36

B. 36 : 25

C. 50 : 41

D. 41 : 50

Answer: C

Solution:

Solution:
Given, radius of sphere $= R$
Distance of particle from centre of Earth $= 3R$
Force between sphere and particle $= F_1$
Radius of cavity $= R/2$
Let mass of sphere $= M$
Mass of sphere with cavity $= M'$
Mass of particle $= m$

Now, $M' = \rho \cdot \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$

By using concept of gravitational force,

$$F_1 = \frac{GMm}{(3R)^2} = \frac{GMm}{9R^2} \ldots \text{(i)}$$

and $F_{cavity} = \frac{GM'm}{(AB)^2} = \frac{(2}{25 \times 8R^2}$

$$= \frac{4GMm}{25} \ldots \text{(ii)}$$

$$\therefore F_2 = F_1 - F_{cavity} = \frac{GMm}{9R^2} - \frac{4}{25 \times 8} \frac{GMm}{R^2}$$

$$= \frac{GMm}{R^2}\left(\frac{1}{9} - \frac{1}{50}\right) = \frac{41}{50 \times 9} \frac{GMm}{R^2}$$

$$\because \frac{F_1}{F_2} = \frac{50}{41}$$

Question75

A planet revolving in elliptical orbit has
I. a constant velocity of revolution
II. has the least velocity when it is nearest to the Sun
III. its areal velocity is directly proportional to its velocity
IV. areal velocity is inversely proportional to its velocity.
V. to follow a trajectory such that the areal velocity is constant.
Choose the correct answer from the options given below.
[26 Feb 2021 Shift 1]

Options:

A. Only I

B. Only IV

C. Only III

D. Only V

Answer: D

Solution:

Solution:
According to Kepler's second law of planetary motion, areal velocity of every planet moving around the sun should remain constant in elliptical orbit.

Question76

A body weights 49N on a spring balance at the North pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator ? (Use, $g = \frac{GM}{R^2} = 9.8 ms^{-2}$ and radius of earth, $R = 6400 km$)

[24 Feb 2021 Shift 2]

Options:

A. 49N

B. 48.83N

C. 49.83N

D. 49.17N

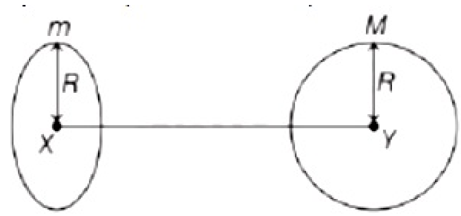
Answer: B

Solution:

Solution:
Given, weight of body at North pole,
$w_p = mg = 49N$
Radius of Earth, $R = 6400 km$
Let weight of body at equator be w_e.
At equator, $g_e = g - R\omega^2$
$\therefore\ w_e = mg_e = m(g - R\omega^2)$
Since, $w_p > w_e \Rightarrow w_e < 49N$
Hence, above condition is satisfied by only option (b).

Question77

Find the gravitational force of attraction between the ring and sphere as shown in the figure, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}R$ is the distance between the centres of a ring (of mass m) and a sphere (of mass M), where both have equal radius R.



[26 Feb 2021 Shift 1]

Options:

A. $\frac{\sqrt{8}}{9} \cdot \frac{GmM}{R}$

B. $\frac{2\sqrt{2}}{3} \cdot \frac{GMm}{R^2}$

C. $\frac{1}{3\sqrt{8}} \cdot \frac{GM\,m}{R^2}$

D. $\frac{\sqrt{8}}{27} \cdot \frac{GmM}{R^2}$

Answer: D

Solution:

Solution:
Given, distance between centre of ring and sphere, $d = \sqrt{8}R$
Since, gravitational field at the axis of ring, $E = \frac{Gmd}{(d^2 + R^2)^{3/2}}$
Here, G is the gravitational constant.
$\Rightarrow E = \frac{GmR\sqrt{8}}{(8R^2 + R^2)^{3/2}} = \frac{GmR\sqrt{8}}{(3R)^3}$
$\Rightarrow E = \frac{GmR\sqrt{8}}{27R^3} = \frac{Gm\sqrt{8}}{27R^2}$
Force between ring and sphere, $F = ME$ . . . (i)
Substituting the value of E in Eq. (i), we get
$F = \frac{\sqrt{8}}{27}\frac{GmM}{R^2}$

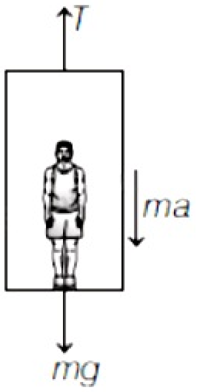
Question78

A person standing on a spring balance inside a stationary lift measures 60kg. The weight of that person, if the lift descends with uniform downward acceleration of 1.8m / s^2 will be .... N. [g = 10m / s^2] [26 Feb 2021 Shift 1]

Answer: 492

Solution:

Solution:
Given, mass of man$(m) = 60\text{kg}$
Downward acceleration of lift, $a = 1.8\text{ms}^{-2}$
Let T be the tension in the rope connected with lift, g be the acceleration due to gravity (10ms^{-2}).
As, lift is moving in downward direction

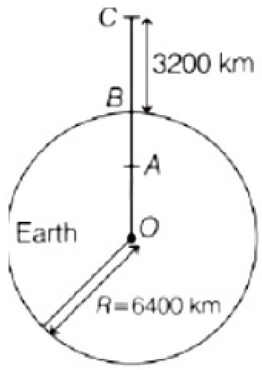


$\therefore\ mg - T = ma$
$\Rightarrow T = mg - ma = m(g - a)$

$= 60(10 - 1.8) = 60 \times 8.2$
$= 492\text{N}$
Hence, the weight of the man during downward acceleration is 492N.

Question79

In the reported figure of Earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the Earth). The value of OA : AB will be x : y. The value of x is ....



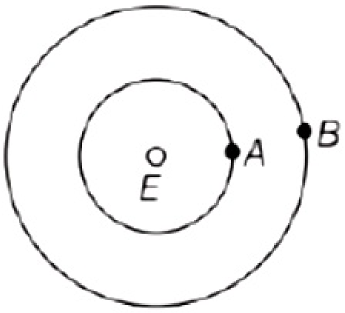
[26 Feb 2021 Shift 2]

Solution:

Given, $g_A = g_C$
Radius of Earth, $R = 6400\text{km}$
Height, $h = 3200\text{km} = \frac{R}{2}$
Since, $g = \frac{GM}{R^2}$
where, G is gravitational constant.
$\therefore\ g_A = \frac{GM\,(OA)}{R^3}$ . . . (i)
and $g_C = \frac{GM}{(OC)^2} = \frac{GM}{R+h}$
$= \frac{GM}{(R + R/2)^2} = \frac{GM}{(3R/2)^2}$ . . . (ii)
According to given information from Eqs. (i) and (ii), we get
$\Rightarrow\ OA = \frac{4}{9}R = \frac{4}{9} \times 6400$
$AB = OB - OA$
and $AB = 6400 - \frac{4}{9} \times 6400$
$\Rightarrow\ = 6400(1 - 4/9) = 6400 \times 5/9$
Now, $OA : AB = \frac{6400 \times 4/9}{5/9 \times 6400} = \frac{4}{5}$
Hence, $x = 4$

Question80

Two satellites A and B of masses 200kg and 400kg are revolving around the Earth at height of 600km and 1600km, respectively. If T_A and T_B are the time periods of A and B respectively, then the value of $T_B - T_A$ is



(Given, radius of Earth = 6400km, mass of Earth = 6×10^{24}kg)
[25 Feb 2021 Shift 1]

Options:

A. 1.33×10^3s

B. 3.33×10^2s

C. 4.24×10^3s

D. 4.24×10^2s

Answer: A

Solution:

Given,
$M_A = 200\text{kg}, M_B = 400\text{kg}, H_A = 600\text{km}, H_B = 1600\text{km}$ and $R_A = R_E + H_A = 6400 + 600 = 7000\text{km}$
$R_B = R_E + H_B = 6400 + 1600 = 8000\text{km}$
Let T_A, T_B, ω_A, ω_B, R_A and R_B be the time period, angular frequencies and radii of satellites A and B, respectively. Force on satellite A, $F_A = m_A\omega_A{}^2R_A = \frac{GM\,m_A}{R_A{}^2}$

$\Rightarrow \omega_A{}^2 = \frac{GM}{R_A{}^3}$

but, $\omega_A = \frac{2\pi}{T_A}$

$\therefore \left(\frac{2\pi}{T_A}\right)^2 = \frac{GM}{R_A{}^3} \Rightarrow T_A = \sqrt{\frac{4\pi^2R_A{}^3}{GM}}$

Similarly, $T_B = \sqrt{\frac{4\pi^2R_B{}^3}{GM}}$

$\therefore T_B - T_A = \sqrt{\frac{4\pi^2}{GM}}\left(\sqrt{R_B{}^3} - \sqrt{R_A{}^3}\right)$

$= \frac{2\pi}{\sqrt{GM}}\left[\sqrt{(8 \times 10^3)^3} - \sqrt{(7 \times 10^3)^3}\right]$

$= \frac{2\pi \times 10^9}{\sqrt{GM}}(8\sqrt{8} - 7\sqrt{7})$

$= \frac{2\pi \times 10^9}{\sqrt{6.67 \times 10^{-11} \times 6 \times 10^{24}}}(4.107)$

$= \frac{2\pi}{\sqrt{4 \times 10^{14}}} \times 10^9(4.107)$

$= \pi \times 10^2 \times 4.107 = 12.9 \times 10^2$s
$= 1.33 \times 10^3$s

Question81

Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.
Assertion (A) When a rod lying freely is heated, no thermal stress is developed in it.
Reason (R) On heating, the length of the rod increases.
In the light of the above statements, choose the correct answer from the options given below
[25 Feb 2021 Shift 1]

Options:

A. Both A and R are true but R is not the correct explanation of A.

B. A is false but R is true

C. A is true but R is false.

D. Both A and R are true and R is the correct explanation of A.

Answer: A

Solution:

Solution:
Thermal stress is defined as the stress, experienced by any rod on heating between two fixed rigid supports. On heating, the size of the rod increases but, if the two ends are free, rod will not experience any stress. i.e, there is no thermal stress will be produced in it.
Hence, option (a) is the correct.

Question82

The initial velocity v_i required to project a body vertically upward from the surface of the Earth to reach a height of 10R, where R is the radius of the Earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be ....
[25 Feb 2021 Shift 2]

Solution:

By using law of conservation of energy,
Energy on the surface of earth ($E_{surface}$) = Energy at height ($h = 10R$)

$$\Rightarrow \frac{-GM\,m}{R} + \frac{1}{2}mv_i^{\,2} = \frac{-GM\,m}{R+10R} + 0 = \frac{-GM\,m}{11R}$$

$$\Rightarrow 1/2mv_i^{\,2} = \frac{11}{11}\frac{GM\,m}{R} - \frac{GM\,m}{11R}$$

$$\Rightarrow \frac{1}{2}mv_i^{\,2} = \frac{10GM\,m}{11R} \Rightarrow v_i^{\,2} = \frac{20GM}{11R}$$

$$v_i = \sqrt{\frac{10}{11}}v_e \left(\because v_e = \sqrt{\frac{2GM}{R}} = \text{ escape velocity}\right)$$

$$\Rightarrow x = 10$$

Question83

Two stars of masses m and 2m at a distance d rotate about their common centre of mass in free space. The period of revolution is : [24 Feb 2021 shift 1]

Options:

A. $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$

B. $2\pi\sqrt{\frac{d^3}{3Gm}}$

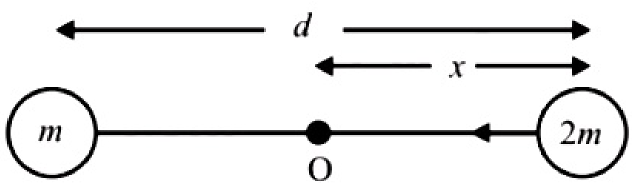
C. $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$

D. $2\pi\sqrt{\frac{3Gm}{d^3}}$

Answer: B

Solution:

Solution:



For point O to be the centre of mass of the system, moment about O should be zero.

$\therefore 2mx = m(d - x)$

$\Rightarrow 3mx = md$

$\Rightarrow x = \frac{d}{3}$ For equilibrium,

$F_{\text{gravitational}} = F_{\text{centripetal}}$

$\therefore F = \frac{G(2m)m}{d^2} = (2m)\omega^2\left(\frac{d}{3}\right)$

$\Rightarrow \frac{Gm}{d^2} = \omega^2\frac{d}{3}$

$\Rightarrow \omega^2 = \frac{3Gm}{d^3}$

$\Rightarrow \omega = \sqrt{\frac{3Gm}{d^3}}$

$\therefore$ Period of revolution,

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{3Gm}}$

Question84

Four identical particles of equal masses 1kg made to move along the circumference of a circle of radius 1m under the action of their own mutual gravitational attraction. The speed of each particle will be : [24 Feb 2021 Shift 1]

Options:

A. $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$

B. $\sqrt{G(1+2\sqrt{2})}$

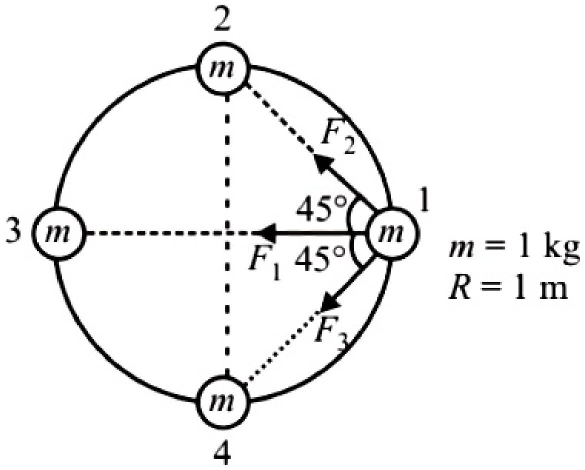
C. $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$

D. $\frac{\sqrt{(1+2\sqrt{2})G}}{2}$

Answer: D

Solution:

Solution:



Gravitational force acting between particle 1 and 3 is

$F_1 = \frac{Gmm}{(2R)^2} = \frac{Gm^2}{4R^2}$

Gravitational force acting between particle 1 and 2 is

$F_2 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$

Gravitational force acting between 1 and 4 is

$F_3 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$x`

Net force towards the centre, $F_{net} = F_1 + F_2\cos 45^\circ + F_3\cos 45^\circ$

$= \frac{Gm^2}{4R^2} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}}$

$= \frac{Gm^2}{R^2}\left(\frac{1}{4} + \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$

$= \frac{Gm^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) = \frac{Gm^2}{4R^2}(1+2\sqrt{2})$

At equilibrium $F_{centre} = F_{centripetal}$

$F_{net} = \frac{Gm^2}{4R^2}(1+2\sqrt{2}) = \frac{mv^2}{R}$

$$\Rightarrow V = \sqrt{\frac{(2\sqrt{2}+1)Gm}{4R}}$$

Put $m = 1kg, R = 1m$, we get

$$\Rightarrow v = \frac{\sqrt{G(1+2\sqrt{2})}}{2}$$

Question85

Consider two satellites S_1 and S_2 with periods of revolution 1hr. and 8hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellites S_2 is [24feb2021shift1]

Options:

A. 8 : 1

B. 1 : 4

C. 2 : 1

D. 1 : 8

Answer: A

Solution:

Solution:

Ratio $\frac{T_1}{T_2} = \frac{1}{8}$

$$\frac{\frac{2\pi}{\omega_1}}{\frac{2\pi}{\omega_2}} = \frac{1}{8} \left(\because T = \frac{2\pi}{\omega}\right)$$

$$\frac{\omega_2}{\omega_1} = \frac{8}{1}$$

Question86

The maximum and minimum distance of a comet from the Sun are 1.6×10^{12}m and 8.0×10^{10}m, respectively. If the speed of the comet at the nearest point is $6 \times 10^4 ms^{-1}$, then the speed at the farthest point is [16 Mar 2021 Shift 1]

Options:

A. $1.5 \times 10^3 m/s$

B. $6.0 \times 10^3 m/s$

C. $3.0 \times 10^3 m/s$

D. $4.5 \times 10^3 m/s$

Answer: C

Solution:

Given, maximum distance of comet from the Sun,
‹ 10^{12}m
Minimum distance of comet from the Sun,
$r_2 = 8.0 \times 10^{10}$m
Speed of the comet at nearest point, $v_2 = 6 \times 10^4 ms^{-1}$
Applying law of conservation of angular momentum,
$mv_1r_1 = mv_2r_2$($\because$ mass of comet will remain same)

$$\Rightarrow V_1 = \frac{V_2 r_2}{r_1}$$

$$= \frac{6 \times 10^4 \times 8 \times 10^{10}}{1.6 \times 10^{12}} = \frac{48 \times 10^{14}}{1.6 \times 10^{12}}$$

$$= 3 \times 10^3 m/s$$

Question87

A geostationary satellite is orbiting around an arbitrary planet P at a height of 11R above the surface of P, R being the radius of P. The time period of another satellite in hours at a height of 2R from the surface of P is P has the time period of 24h.
[17 Mar 2021 Shift 2]

Options:

A. $6\sqrt{2}$

B. $\frac{6}{\sqrt{2}}$

C. 3

D. 5

Answer: C

Solution:

Solution:
Given
Height of satellite from planet's surface, $h = 11R$
So, the total distance from the centre of planet $= 11R + R = 12R$
Time period of planet $= 24h$
Similarly, the total distance of second satellite from the centre of planet $= 2R + R = 3R$
Using the Kepler's law
$T^2 \propto R^3$

$$\Rightarrow \left(\frac{T_1}{T_2}\right) = \left(\frac{R_1}{R_2}\right)^{3/2}$$

Substituting the values in the above equation, we get

$$\left(\frac{24}{T}\right) = \left(\frac{12R}{3R}\right)^{3/2}$$

$\therefore$ The time period of another satellite is 3h.

Question88

The time period of a satellite in a circular orbit of radius R is T . The period of another satellite in a circular orbit of radius 9R is [18 Mar 2021 Shift 1]

Options:

A. 9T

B. 27T

C. 12T

D. 3T

Answer: B

Solution:

Solution:
According to Kepler's third law,
$T^2 \propto R^3$
$\Rightarrow \frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2}$
$\Rightarrow \frac{T_2}{T} = \left(\frac{9R}{R}\right)^{3/2}$ $[\because R_1, = R; R_2, = 9R]$
$\Rightarrow T_2 = 27T$

Question89

The angular momentum of a planet of mass M moving around the Sun in an elliptical orbit is L. The magnitude of the areal velocity of the planet is [18 Mar 2021 Shift 2]

Options:

A. $\frac{4L}{M}$

B. $\frac{L}{M}$

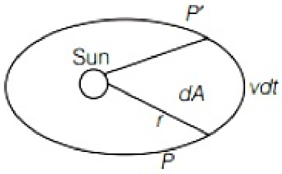
C. $\frac{2L}{M}$

D. $\frac{L}{2M}$

Answer: D

Solution:

Solution:
According to the Kepler's second law,
frac d A d t = constant



Area covered from P to P' is d A, then

$dA = \frac{1}{2}r^2 d\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt}$

$\Rightarrow \frac{dA}{dt} = \frac{1}{2}r^2\omega$...(i)

Angular momentum,

$L = Mvr = M(\omega r)r = M\omega r^2$

$\Rightarrow \omega = \frac{L}{mr^2}$

∴ From Eqs. (i) and (ii), we get

$\frac{dA}{dt} = \frac{1}{2}r^2\left(\frac{L}{Mr^2}\right)$

$\Rightarrow \frac{dA}{dt} = \frac{L}{2M}$

Question90

If the angular velocity of Earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately (Take $2g = 10ms^{-2}$, the radius of Earth, $R = 6400 \times 10^3 m$, take $\pi = 3.14$) [18 Mar 2021 Shift 2]

Options:

A. 60min

B. does not change

C. 1200min

D. 84min

Answer: D

Solution:

Solution:

Condition of weightlessness, $g_{\text{equator}} = 0$

$g_{\text{equazar}} = g - \omega^2 R$

The radius of the Earth, $R = 6400km = 6.4 \times 10^6 m$

$\Rightarrow \omega = \sqrt{\frac{g}{R}}$

$\omega = \sqrt{\frac{10}{6.4 \times 10^6 m}} \Rightarrow \omega = \frac{1}{800}rad/s$

The duration of the day,

$T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{\frac{1}{800}}$

$\Rightarrow T = 5024s \Rightarrow T = \frac{5024}{60}min$

$\Rightarrow T = 84min$

Hence, the duration of the day would be 84 min.

Question91

A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height h is ______ S.
[22 Jul 2021 Shift 2]

Options:

A. $\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

B. $\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

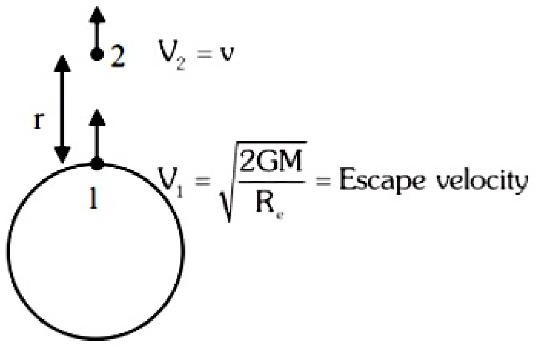
C. $\frac{1}{3}\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

D. $\frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

Answer: D

Solution:

Solution:



Applying energy conservation from (1) to (2)

$\frac{1}{2}m.\left(\frac{2GM}{R_e}\right)-\frac{GM\,m}{R_e} = \frac{1}{2}mv^2-\frac{GM\,m}{R+r}$

$\Rightarrow\frac{1}{2}mv^2 = \frac{GM\,m}{R+r}$

$v = \sqrt{\frac{2GM}{R+r}} = \frac{dr}{dt}$

$\sqrt{2GM}\int_0^t dt = \int_{R_e}^{R_e+h}(\sqrt{R+r})dr$

$\sqrt{2GM}\,.\,t = \frac{2}{3}[(R+r)^{3/2}]_{R_e}^{R_e+h}$

$t = \frac{2}{3}\sqrt{\frac{R_e^{\,3}}{2GM}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

$\frac{GM}{R_e^{\,2}} = g$

$t = \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

Question92

If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$, where x is
(Round off to the nearest integer) (M is the mass of earth, R is the radius of earth and G is the gravitational constant.)
[16 Mar 2021 Shift 2]

Solution:

We know that binding energy of earth,
$BE = -\frac{3}{5}\frac{GM^2}{R}$
∴ Energy required to break the earth into pieces
$= -BE = \frac{3}{5}\frac{GM^2}{R}$ ....(i)
According to question, the amount of energy that needs to be supplied is $\frac{x}{5}\frac{GM^2}{R}$.
Comparing it with value in Eq. (i), we get,
$x = 3$

Question93

The radius in kilometre to which the present radius of Earth (R = 6400km) to be compressed so that the escape velocity is increased 10 times is
[17 Mar 2021 Shift 1]

Answer: 64

Solution:

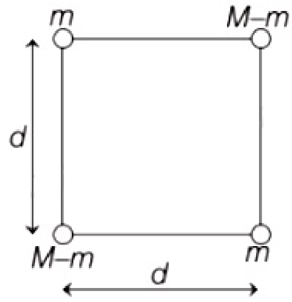
We know that, $v_e = \sqrt{\frac{2Gm}{R_1}}$ ...(i)
where, $v_e =$ escape velocity,
$G =$ gravitational constant,
$R =$ radius of earth
and $m =$ mass of the body.
Now, the escape velocity is increased to 10 times,
$\Rightarrow 10v_e = \sqrt{\frac{2Gm}{R_2}}$...(ii)
Dividing Eq. (ii) by Eq. (i), we get

$\Rightarrow 10 = \sqrt{\frac{R_1}{R_2}} \Rightarrow 100 = \frac{R_1}{R_2}$

$\Rightarrow R_2 = \frac{R_1}{100} = \frac{6400}{100} \Rightarrow R_2 = 64\text{km}$

Question94

A body of mass 2M splits into four masses {m, M − m, m, M − m}, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$ for which, the gravitational potential energy of the system becomes maximum is x : 1 . The value of x is......... .



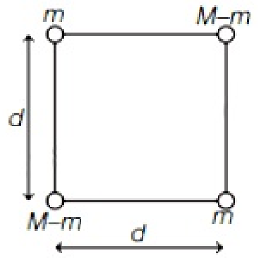
[27 Aug 2021 Shift 1]

Solution:

Given, total mass of body is 2M.

Potential energy is maximum at $\frac{M}{m} = \frac{x}{1}$

The arrangement of masses to form a square is shown in diagram.



The gravitational potential energy of a body is given

$U = -\frac{GMm}{r}$

For the given system, the potential energy will be

$U_T = -\frac{Gm(M-m)}{d} - \frac{Gm(M-m)}{d} - \frac{Gm(M-m)}{d} - \frac{Gm(M-m)}{d} - \frac{Gm^2}{(\sqrt{2}d)^2} - \frac{G(M-m)^2}{(\sqrt{2}d)^2}$

$U_T = -\frac{4Gm(M-m)}{d} - \frac{Gm^2}{\sqrt{2}d} - \frac{G(M-m)^2}{\sqrt{2}d}$

For maximum potential energy,

$\frac{dU_T}{dm} = 0$

$-\frac{4G}{d}[M-2m] - \frac{G}{\sqrt{2}d}[2m]$

$-\frac{G}{\sqrt{2}d}[2(M-m)\times -1] = 0$

$\Rightarrow 4M - 8m + \sqrt{2}m = \sqrt{2}(M-m)$

$(4-\sqrt{2})M = (8-2\sqrt{2})m$

$\frac{M}{m} = \frac{2(4-\sqrt{2})}{4-\sqrt{2}} = 2$

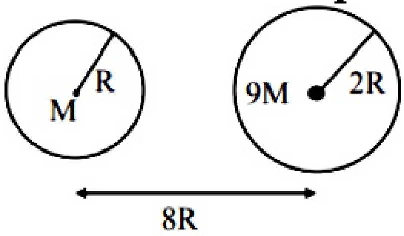
Comparing it with the given value, we get x = 2

Thus, potential energy will be maximum when x = 2.

Question95

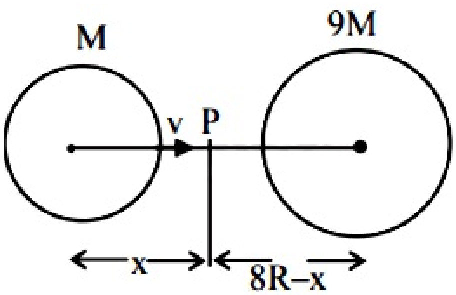
Suppose two planets (spherical in shape) of radii R and 2R, but mass M and 9 M respectively have a centre to centre separation 8 R as shown in the figure. A satellite of mass 'm' is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed 'v' required for the satellite to reach the surface of the second planet is $\sqrt{\frac{a}{7}\frac{GM}{R}}$ then the value of 'a' is _______.

[Given : The two planets are fixed in their position]



[27 Jul 2021 Shift 1]

Solution:



Assume that at a distance x from the planet of mass M , the net gravitational field becomes zero.

$\therefore \frac{GM}{x^2} = \frac{G \times 9M}{(8R-x)^2}$

$\Rightarrow \frac{1}{x^2} = \frac{9}{(8R-x)^2}$

$\Rightarrow \left(\frac{1}{x}\right) = \left(\frac{3}{8R-x}\right)^2 \Rightarrow \frac{1}{x} = \frac{3}{8R-x}$

$\Rightarrow 3x = 8R - x$

$\Rightarrow 4x = 8R$

$\Rightarrow x = 2R \quad \cdots\cdots (i)$

Now, a satellite should be projected in such a way that its covers a minimum distance of 2R .

$\therefore \frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{G(9M)m}{7R}$

$= \frac{-GMm}{2R} - \frac{G(9M)m}{6R}$

where, m is the mass of satellite.

$\Rightarrow \frac{1}{2}v^2 = \frac{2GM}{7R} \Rightarrow v = \sqrt{\frac{4}{7}\frac{GM}{R}}$ ...... (ii)

According to question, the minimum speed v required for the satellite to reach the surface of the second planet is $\sqrt{\frac{aGM}{7R}}$. So, on comparing it with Eq. (ii), we can write

a = 4

Question96

A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02 R. The percentage difference in the time periods of the two satellites is : [20 Jul 2021 Shift 2]

Options:

A. 1.5

B. 2.0

C. 0.7

D. 3.0

Answer: D

Solution:

$T^2 \propto R^3$

$T = kR^{3/2}$

$\frac{dT}{T} = \frac{3}{2}\frac{dR}{R}$

$= \frac{3}{2} \times 0.02 = 0.03$

% Change $= 3\%$

Question97

The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital radius of 9.0×10^3km. Find the mass of Mars.

$\left\{ \text{Given } \frac{4\pi^2}{G} = 6 \times 10^{11} N^{-1} m^{-2} kg^2 \right\}$

[27 Jul 2021 Shift 2]

Options:

A. 5.96×10^{19}kg

B. 3.25×10^{21}kg

C. 7.02×10^{25}kg

D. 6.00×10^{23}kg

Answer: D

Solution:

Option D is correct

$$GM \quad r^3$$

$$M = \frac{4\pi^2}{G} \cdot \frac{r^3}{T^2}$$

by putting values

$M = 6 \times 10^{23}$

Question98

A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50m. If the gravitational potential at a point, 25m from the centre is V kg / m. The value of V is
[27 Aug 2021 Shift 2]

Options:

A. -60 G

B. +2 G

C. -20 G

D. -4 G

Answer: D

Solution:

Solution:

Question99

If R_E be the radius of Earth, then the ratio between the acceleration due to gravity at a depth r below and a height r above the Earth surface is (Given, $r < R_E$)
[31 Aug 2021 Shift 2]

Options:

A. $1 - \frac{r}{R_E} - \frac{r^2}{{R_E}^2} - \frac{r^3}{{R_E}^3}$

B. $1 + \frac{r}{R_E} + \frac{r^2}{{R_E}^2} + \frac{r^3}{{R_E}^3}$

C. $1+\frac{r}{R_E}-\frac{r^2}{R_E{}^2}+\frac{r^3}{R_E{}^3}$

D. $1+\frac{r}{R_E}-\frac{r^2}{R_E{}^2}-\frac{r^3}{R_E{}^3}$

Answer: D

Solution:

Solution:
Given that, radius of Earth $= R_E$
Height above Earth's surface, h = r
Depth below Earth surface, d = r
We know that, acceleration due to gravity at height (r) and depth (r) is (for $r < R_E$)

$g_h = g\left(\frac{R_E}{R_E+h}\right)^2 = g\left(\frac{R_E}{R_E+r}\right)^2$ ...(i)

and$g_d = g\left(\frac{R_E-d}{R_E}\right) = g\left(\frac{R_E-r}{R_E}\right)$ ...(ii)

$\therefore \frac{g_d}{g_h} = \frac{g\left(\frac{R_E-r}{R_E}\right)}{g\left(\frac{R_E}{R_E+r}\right)^2}$ [from Eqs. (i) and (ii)]

$\Rightarrow \frac{g_d}{g_h} = \frac{(R_E-r)(R_E+r)^2}{R^3} = \frac{R_E{}^3+R_E{}^2r-R_Er^2-r^3}{R_E{}^3}$

$1+\frac{r}{R_E}-\frac{r^2}{R_E{}^2}-\frac{r^3}{R_E{}^3}$

Question100

Inside a uniform spherical shell
I. the gravitational field is zero.
II. the gravitational potential is zero.
III. the gravitational field is same everywhere
IV. the gravitation potential is same everywhere.
V. All of the above
Choose the most appropriate answer from the options given below.
[26 Aug 2021 Shift 1]

Options:

A. I, III and IV

B. Only V

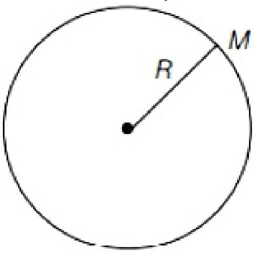
C. I, II and III

D. II, III and IV

Answer: A

Solution:

The uniform spherical shell is shown in the figure below



Inside the shell, there is no mass, so by applying Gauss's law for gravitation
$\oint g \cdot d a=-4 \pi G M_{\text {enclosed }}$
where, g = gravitational field intensity.
da = area enclosed
G = gravitational constant
and $M_{\text {enclosed }}=$ mass enclosed .
As, $M_{\text {enclosed }}=0$
So, gravitational field intensity is zero inside the shell, i.e. g = 0 ...(i)
We also know that,
$g=-\frac{d V}{d r}$
where, dV is change in potential due to gravity
⇒ V = constant [using Eq. (i)]
So, gravitational field is zero everywhere inside the shell and
gravitational potential (V) is constant everywhere.

Question101

The masses and radii of the Earth and Moon are (m_1, R_1) and (m_2, R_2), respectively. Their centres are at a distance r apart. Find the minimum escape velocity for a particle of mass m to be projected from the middle of these two masses.
[31 Aug 2021 Shift 1]

Options:

A. $v=\frac{1}{2} \sqrt{\frac{4 G(m_1+m_2)}{r}}$

B. $v=\sqrt{\frac{4 G(m_1+m_2)}{r}}$

C. $v=\frac{1}{2} \sqrt{\frac{2 G(m_1+m_2)}{r}}$

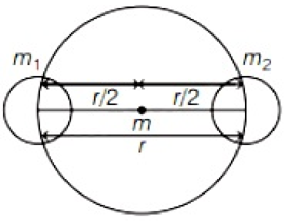
D. $v=\frac{\sqrt{2} G(m_1+m_2)}{r}$

Answer: B

Solution:

Solution:
Given, masses and radii of Earth and Moon are (m_1, R_1), (m_2, R_2)
and separation between their centre of mass = r
If v be the minimum escape velocity of particle of mass m, then by using law of conservation of energy,
Initial energy = Final energy = 0

$\frac{1}{2}mv^2 - \frac{Gm_1m}{\frac{r}{2}} - \frac{Gm_2m}{\frac{r}{2}} = 0$



$\Rightarrow \frac{1}{2}mv^2 = \frac{2Gm}{r}(m_1 + m_2)$

$\Rightarrow v = \sqrt{\frac{4G}{r}(m_1 + m_2)}$

Question102

Two satellites revolve around a planet in coplanar circular orbits in anti-clockwise direction. Their period of revolutions are 1h and 8h, respectively. The radius of the orbit of nearer satellite is 2×10^3 km. The angular speed of thefarther satellite as observed from the nearer satellite at the instant when both the satellites are closest is $\frac{\pi}{x}$ rad h^{-1}, where x is

[1 Sep 2021 Shift 2]

Solution:

Given, the time period of the first satellite, $T_1 = 1h$
The time period of the second satellite, $T_2 = 8$ h
The radius of the orbit of nearer satellite, $R_1 = 2000$ kmApplying the Kepler's third law,
$T^2 \propto R^3$

$\left(\frac{T_1}{T_2}\right) = \left(\frac{R_1}{R_2}\right)^3$

$\Rightarrow \frac{2000}{R_2} = \left(\frac{1}{8}\right)^{\frac{2}{3}}$

$\Rightarrow R_2 = 8000$ km
As we know the relation between the angular speed and timeperiod

$\omega = \frac{\theta}{T}$ So, the angular speed of the nearer satellite to the orbit,

$\omega_1 = \frac{2\pi}{1}$ rad / h

and the angular speed of the farther satellite to the orbit,

$\omega_2 = \frac{2\pi}{8} = \frac{\pi}{4}$ rad / h

The speed of the nearer satellite to orbit,
$v_1 = \omega_1 R_1 = (2\pi) \times 2 \times 10^3$ km / h
The speed of the farther satellite to the orbit
$V_2 = \omega_2 R_2$
$= \left(\frac{\pi}{4}\right) \times 8000$

$= \pi \times 2 \times 10^3 \text{ km / h}$

Thus, the relative angular speed of the nearer satellite to the farthersatellite,

$$\omega = \frac{V_1 - V_2}{R_1 - R_2}$$

$$= \frac{2\pi \times 2 \times 10^3 - \pi \times 2 \times 10^3}{8000 - 2000}$$

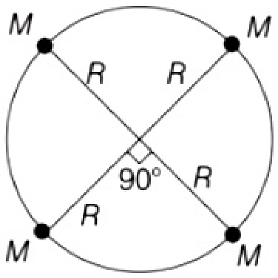
$$= \frac{2\pi \times 10^3}{6000} = \frac{\pi}{3} \text{ rad / h}$$

Comparing with, $\omega = \frac{\pi}{x}$

The value of x = 3.

Question103

Four particles each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction as shown in figure. The speed of each particle is



[1 Sep 2021 Shift 2]

Options:

A. $\frac{1}{2}\sqrt{\frac{GM}{R(2\sqrt{2}+1)}}$

B. $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$

C. $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}-1)}$

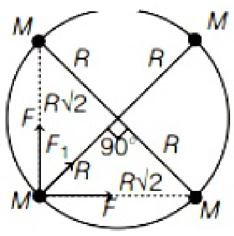
D. $\sqrt{\frac{GM}{R}}$

Answer: B

Solution:

Let us consider the gravitational force acting on each mass M by adjacent particles be F.

avitational force acting on each mass M diagonally be F_1



The net force, $F_{net} = \frac{Mv^2}{R}$

Along the centre of circle,

$$\sqrt{2}F + F_1 = \frac{Mv^2}{R}$$

$$\sqrt{2}\left(\frac{GMM}{(\sqrt{2}R)^2}\right) + \left(\frac{GMM}{(2R)^2}\right) = \frac{Mv^2}{R}$$

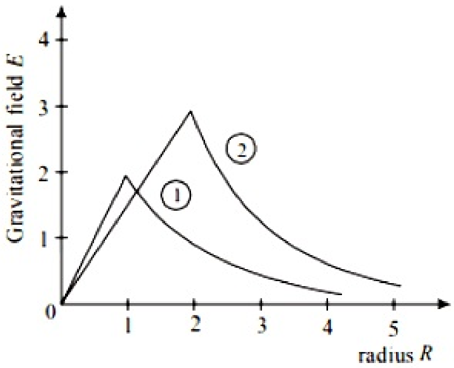
$$\Rightarrow \frac{GM}{R^2}\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = \frac{v^2}{R}$$

$$\Rightarrow v = \frac{1}{2}\sqrt{\frac{GM(2\sqrt{2}+1)}{R}}$$

Hence, the speed of each particle is $\frac{1}{2}\sqrt{\frac{GM(2\sqrt{2}+1)}{R}}$.

Question104

Consider two solid spheres of radii $R_1 = 1m$, $R_2 = 2m$ and masses M_1 and M_2, respectively. The gravitational field due to sphere (1) and (2) are shown. The value of $\frac{m_1}{m_2}$ is:



[8 Jan. 2020 I]

Options:

A. $\frac{2}{3}$

B. $\frac{1}{6}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

Answer: B

Solution:

Gravitation field at the surface

$E = \frac{\quad}{r^2}$

$\therefore E_1 = \frac{Gm_1}{r_1^2}$ and $E_2 = \frac{Gm_2}{r_2^2}$

From the diagram given in question,

$\frac{E_1}{E_2} = \frac{2}{3}(r_1 = 1m, R_2 = 2m$ given)

$\therefore \frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2 \left(\frac{m_1}{m_2}\right) \Rightarrow \frac{2}{3} = \left(\frac{2}{1}\right)^2 \left(\frac{m_1}{m_2}\right)$

$\Rightarrow \left(\frac{m_1}{m_2}\right) = \frac{1}{6}$

Question105

An asteroid is moving directly towards the centre of the earth. When at a distance of 10 R (R is the radius of the earth) from the earths centre, it has a speed of 12 km/s.
Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/ s)? Give your answer to the nearest integer in kilometer/s _____.
[NA 8 Jan. 2020 II]

Solution:

Using law of conservation of energy
Total energy at height 10 R = total energy at earth

$$-\frac{GM_E m}{10R} + \frac{1}{2}mV_0^2 = -\frac{GM_E m}{R} + \frac{1}{2}mV^2$$

[$\because$ Gravitational potential energy $= -\frac{GMm}{r}$]

$$\Rightarrow \frac{GM_E}{R}\left(1 - \frac{1}{10}\right) + \frac{V_0^2}{2} = \frac{V^2}{2}$$

$$\Rightarrow V = \sqrt{V_0^2 + \frac{9}{5}gR} \approx 16 km/s$$

[$\because V = 12 km/s$ given]

Question106

A box weighs 196 N on a spring balance at the north pole.
Its weight recorded on the same balance if it is shifted to the equator is close to (Take g = 10ms^{-2} at the north pole and the radius of the earth = 6400 km):
[7 Jan. 2020 II]

Options:

A. 195.66 N

B. 194.32 N

C. 194.66 N

D. 195.32 N

Answer: D

Solution:

Solution:
Weight at pole, $w = mg = 196N$
$\Rightarrow quad\, m = 19.6kg$
Weight at equator, $w' = mg' = m(g - \omega^2 R)$
$= 19.6\left[10 - \left(\frac{2\pi}{24 \times 3600}\right)^2 \times 6400 \times 10^3\right] N \quad \left(\because \omega = \frac{2\pi}{T}\right)$
$= 19.6[10 - 0.034] = 195.33N$

Question107

A body A of mass m is moving in a circular orbit of radius Rabout a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ or A. The collision is completely inelastic. Then, the combined body:
[9 Jan. 2020 I]

Options:

A. continues to move in a circular orbit

B. Escapes from the Planet's Gravitational field

C. Falls vertically downwards towards the planet

D. starts moving in an elliptical orbit around the planet

Answer: D

Solution:

Solution:
From law of conservation of momentum, $\vec{p}_i = \vec{p}_f$
$m_1 u_1 + m_2 u_2 = M V_f$
$\Rightarrow v_f = \frac{\left(mv + \frac{mv}{4}\right)}{3m2} = \frac{5v}{6}$
Clearly, $v_f < v_i \therefore$ Path will be elliptical

Question108

Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is:
[9 Jan. 2020 II]

Options:

A. 4

B. 1

C. 2

D. 3

Answer: A

Solution:

Solution:

Escape velocity of the planet A is $V_A = \sqrt{\frac{2GM_A}{R_A}}$

where M_A and R_A be the mass and radius of the planet A

According to given problem

$M_B = \frac{M_A}{2}, R_B = \frac{R_A}{2}$

$\therefore V_B = \sqrt{\frac{2G\frac{M_A}{2}}{\frac{R_A}{2}}} \therefore \frac{V_A}{V_B} = \sqrt{\frac{\frac{2GM_A}{R_A}}{\frac{2GM_A/2}{R_A/2}}} = \frac{n}{4} = 1$

$\Rightarrow n = 4$

Question109

A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R(R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth):
[7 Jan. 2020 I]

Options:

A. $\frac{m}{20}\left(u^2 + \frac{113}{200}\frac{GM}{R}\right)$

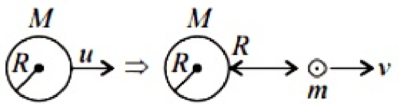
B. $5m\left(u^2 - \frac{119}{200}\frac{GM}{R}\right)$

C. $\frac{3m}{8}\left(u + \sqrt{\frac{5GM}{6R}}\right)^2$

D. $\frac{m}{20}\left(u - \sqrt{\frac{2GM}{3R}}\right)^2$

Answer: B

Solution:



$$\frac{1}{2}mu^2 + \frac{-GM\,m}{R} = \frac{1}{2}mv^2 + \frac{-GM\,m}{2R}$$

$$\Rightarrow \frac{1}{2}m(v^2 - u^2) = \frac{-GM\,m}{2R}$$

$$\Rightarrow V = \sqrt{V = u^2\frac{GM}{R}} \;.......(i)$$

$$v_0 = \sqrt{\frac{GM}{2R}} \therefore v_{rad} = \frac{m \times v}{\left(\frac{m}{10}\right)} = 10v$$

Ejecting a rocket of mass $\frac{m}{10}$

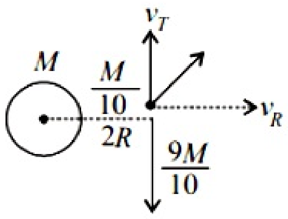
$$\therefore \frac{9m}{10} \times \sqrt{\frac{GM}{2R}} = \frac{m}{10} \times v_\tau \Rightarrow V_\tau{}^2 = 81\frac{GM}{2R}$$

Kinetic energy of rocket,

$$KE_{rocket} = \frac{1}{2}\frac{M}{10}(V_T{}^2 + V_r{}^2)$$

$$= \frac{1}{2} \times \frac{m}{10} \times \left(\left(u^2 - \frac{GM}{R}\right) 100 + 81\frac{GM}{R} \right)$$

$$= \frac{m}{20} \times 100 \left(u^2 - \frac{GM}{R} + \frac{81}{200}\frac{GM}{R}\right)$$



$$= 5m\left(u^2 - \frac{119}{200}\frac{GM}{R}\right)$$

Question110

The mass density of a spherical galaxy varies as frac K r over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as: [2 Sep. 2020 (I)]

Options:

A. $T^2 \propto R$

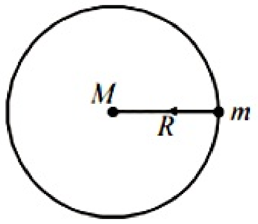
B. $T^2 \propto R^3$

C. $T^2 \propto \frac{1}{R^3}$

D. $T \propto R$

Answer: A

Solution:



According to question, mass density of a spherical galaxy varies as $\frac{k}{r}$

Mass, $M = \int \rho d V$

$\Rightarrow M = \int_{0}^{r=R_0} \frac{k}{r} 4\pi r^2 d r$

$\Rightarrow M = 4\pi k \int_{0}^{R_0} r d r$

or, $M = \frac{4\pi k R_0{}^2}{2} = 2\pi k R^2$

$F_G = \frac{GM m}{R_0{}^2} = m\omega_0{}^2 R (= F_C)$

$\Rightarrow \frac{G\frac{4\pi k R^2}{2}}{R^2} = \omega_0{}^2 R \Rightarrow \omega_0 = \sqrt{\frac{2\pi K G}{R}} \left(\because \omega = \frac{2\pi}{T}\right)$

$\therefore T = \frac{2\pi}{\omega_0} = \frac{2\pi\sqrt{R}}{\sqrt{2\pi K G}} = \sqrt{\frac{2\pi R}{K G}} \Rightarrow T^2 = \frac{2\pi R}{K G}$

$\because 2\pi$, K and G are constants

$\therefore T^2 \propto R$

Question111

A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is :

[3 Sep. 2020 (I)]

Options:

A. $4R_e$

B. $2.5R_e$

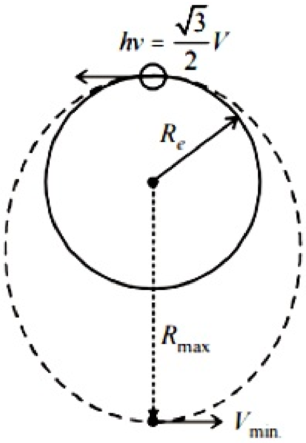
C. $3R_e$

D. $2R_e$

Answer: C

Solution:

Solution:



Orbital velocity, $V_0 = \sqrt{\frac{GM}{R_e}}$

From energy conversation,

$-\frac{GM\,m}{R_e} + \frac{1}{2}m\left(\sqrt{\frac{3}{2}}V\right)^2 = \frac{GM\,m}{R_{max}} + \frac{1}{2}mV_{min}^{\ 2}$ ...(1)

From angular momentum conversation

$\sqrt{\frac{3}{2}}V\,R_e = V_{min}R_{max}$ ...(2)

Solving equation (1) and (2) we get,

$R_{max} = 3R_e$

Question112

A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is :
[4 Sep. 2020 (II)]

Options:

A. $\frac{1}{\sqrt{2}}$

B. 2

C. 1

D. $\sqrt{2}$

Answer: A

Solution:

Solution:
Orbital speed of the body when it revolves very close to the surface of planet

$V_0 = \sqrt{\frac{GM}{R}}$ ...(i)

Here, G = gravitational constant
Escape speed from the surface of planet

$V_e = \sqrt{\frac{2GM}{R}}$ ...(ii)

Dividing (i) by (ii), we have

$$\frac{V_0}{V_e} = \frac{\sqrt{\frac{GM}{R}}}{\sqrt{\frac{2GM}{R}}} = \frac{1}{\sqrt{2}}$$

Question113

The value of acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$(R = radius of the earth) from the surface of the earth. It is again equal to g_1 and a depth d below the sur-face of the earth. The ratio $\left(\frac{d}{R}\right)$ equals: [5 Sep. 2020 (I)]

Options:

A. $\frac{4}{9}$

B. $\frac{5}{9}$

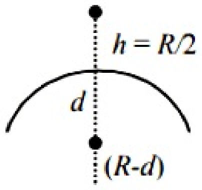
C. $\frac{1}{3}$

D. $\frac{7}{9}$

Answer: B

Solution:

Solution:
According to question, $g_h = g_d = g_1$



$g_h = \frac{GM}{\left(R + \frac{R}{2}\right)^2}$ and $g_d = \frac{GM(R-d)}{R^3}$

$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GM(R-d)}{R^3} \Rightarrow \frac{4}{9} = \frac{(R-d)}{R}$

$\Rightarrow 4R = 9R - 9d \Rightarrow 5R = 9d$

$\therefore \frac{d}{R} = \frac{5}{9}$

Question114

The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same,

then h is: ($h << R$, where R is the radius of the earth)
[5 Sep. 2020 (II)]

Options:

A. $\frac{R^2\omega^2}{2g}$

B. $\frac{R^2\omega^2}{g}$

C. $\frac{R^2\omega^2}{4g}$

D. $\frac{R^2\omega^2}{8g}$

Answer: B

Solution:

Solution:
Value of g at equator, $g_A = g. - R\omega^2$
Value of g at height h above the pole,
$g_B = g \cdot \left(1 - \frac{2h}{R}\right)$
As object is weighed equally at the equator and poles, it means g is same at these places.
$g_A = g_B$
$\Rightarrow g - R\omega^2 = g\left(1 - \frac{2h}{R}\right)$
$\Rightarrow R\omega^2 = \frac{2gh}{R} \Rightarrow h = \frac{R^2\omega^2}{2g}$

Question115

The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected) :
[2 Sep. 2020 (II)]

Options:

A. $\frac{\sqrt{5}}{2}R - R$

B. $\frac{R}{2}$

C. $\frac{\sqrt{5}R - R}{2}$

D. $\frac{\sqrt{3}R - R}{2}$

Answer: C

Solution:

The acceleration due to gravity at a height h is given by

$g = \frac{GM}{(R+h)^2}$

Here, G = gravitation constant

M = mass of earth

The acceleration due to gravity at depth h is

$g' = \frac{GM}{R^2}\left(1 - \frac{h}{R}\right)$

Given, $g = g'$

$\therefore \frac{GM}{(R+h)^2} = \frac{GM}{R^2}\left(1 - \frac{h}{R}\right)$

$\therefore R^3 = (R+h)^2(R-h) = (R^2 + h^2 + 2hR)(R-h) \Rightarrow R^3 = R^3 + h^2R + 2hR^2 - R^2h - h^3 - 2h^2R$

$\Rightarrow h^3 + h^2(2R - R) - R^2h = 0$

$\Rightarrow h^3 + h^2R - R^2h = 0$

$\Rightarrow h^2 + hR - R^2 = 0$

$\Rightarrow h = \frac{-R \pm \sqrt{R^2 + 4(1)R^2}}{2}$

$= \frac{-R + \sqrt{5}R}{2} = \frac{(\sqrt{5} - 1)}{2}R$

Question116

Two planets have masses M and 16 M and their radii are a and 2a, respectively. The separation between the centres of the planets is 10a. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach the surface of smaller planet, the minimum firing speed needed is :
[6 Sep. 2020 (II)]

Options:

A. $2\sqrt{\frac{GM}{a}}$

B. $4\sqrt{\frac{GM}{a}}$

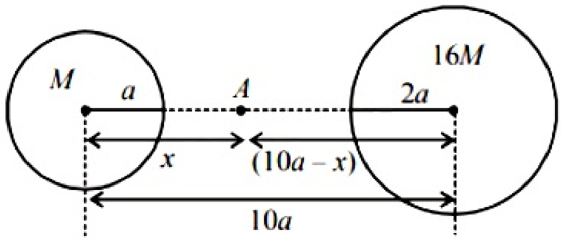
C. $\sqrt{\frac{GM^2}{ma}}$

D. $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

Answer: D

Solution:

Solution:



Let A be the point where gravitation field of both planets cancel each other i.e. zero.

$\frac{GM}{x^2} = \frac{G(16M)}{(10a-x)^2}$

$\Rightarrow \frac{1}{x} = \frac{4}{(10a-x)} \Rightarrow 4x = 10a - x \Rightarrow x = 2a$ ... (i)

Using conservation of energy, we have

$-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE = -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$

$KE = GMm\left[\frac{1}{8a} + 162a - \frac{1}{2a} - \frac{16}{8a}\right]$

$\Rightarrow KE = GMm\left[\frac{1+64-4-16}{8a}\right]$

$\Rightarrow \frac{1}{2}mv^2 = GMm\left[\frac{45}{8a}\right] \Rightarrow v = \sqrt{\frac{90GM}{8a}}$

$\Rightarrow v = \frac{3}{2}\sqrt{\frac{5GM}{a}}$

Question117

On the x-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x -direction. Th magnitude of gravitational potential on the x -axis at a distance x, taking its value to be zero at infinity, is:
[4 Sep. 2020 (I)]

Options:

A. $\frac{A}{(x^2+a^2)^{1/2}}$

B. $\frac{A}{(x^2+a^2)^{3/2}}$

C. $A(x^2+a^2)^{1/2}$

D. $A(x^2+a^2)^{3/2}$

Answer: A

Solution:

Solution:
Given : Gravitational field,

$E_G = \frac{Ax}{(x^2+a^2)^{3/2}}, V_\infty = 0$

$\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G \cdot \vec{d}_x$

$\Rightarrow V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2+a^2)^{3/2}} dx$

$\therefore V_x = \frac{A}{(x^2+a^2)^{1/2}} - 0 = \frac{A}{(x^2+a^2)^{1/2}}$

Question118

The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at:
[3 Sep. 2020 (II)]

Options:

A. $r = \sqrt{\frac{3}{4}}R$

B. $r = R$

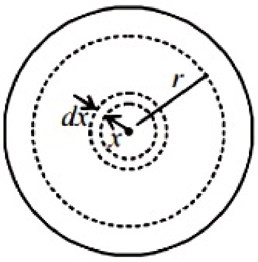
C. $r = \frac{1}{\sqrt{3}}R$

D. $r = \sqrt{\frac{5}{9}}R$

Answer: D

Solution:

Solution:



Mass of small element of planet of radius x and thickness dx.

$dm = \rho \times 4\pi x^2 dx = \rho_0 \left(1 - \frac{x^2}{R^2}\right) \times 4\pi x^2 dx$

Mass of the planet

$M = 4\pi\rho_0 \int_0^r \left(x^2 - \frac{x^4}{R^2}\right) dx$

$\Rightarrow M = 4\pi\rho_0 \left|\frac{r^3}{3} - \frac{r^5}{5R^2}\right|$

Gravitational field,

$E = \frac{GM}{r^2} = Gr^2 \times 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$

$\Rightarrow E = 4\pi G\rho_0 \left(\frac{r}{3} - \frac{r^3}{5R^2}\right)$

E is maximum when $\frac{dE}{dr} = 0$

$\Rightarrow \frac{dE}{dr} = 4\pi G\rho_0 \left(\frac{1}{3} - \frac{3r^2}{5R^2}\right) = 0$

$\Rightarrow r = \frac{\sqrt{5}}{3}R$

Question119

A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is : [NA 6 Sep. 2020 (I)]

Options:

A. 1 : 6

B. 1 : 3

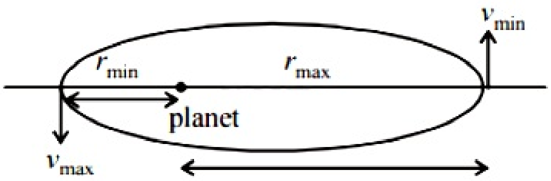
C. 1 : 2

D. 3 : 4

Answer: A

Solution:

Solution:
By angular momentum conservation
$r_{min} v_{max} = m r_{max} v_{min}$



Given, $v_{min} = \frac{v_{max}}{6}$

$\therefore \frac{r_{min}}{r_{max}} = \frac{v_{min}}{v_{max}} = \frac{1}{6}$

Question120

If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is: [9 Jan. 2019 I]

Options:

A. $\frac{L}{m}$

B. $\frac{4L}{m}$

C. $\frac{L}{2m}$

D. $\frac{2L}{m}$

Answer: C

Solution:

Solution:
Areal velocity; $\frac{dA}{dt}$
$dA = \frac{1}{2}r^2 d\theta$
$\Rightarrow \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega$
Also, $L = mvr = mr^2\omega$
$\therefore \frac{dA}{dt} = \frac{1}{2}\frac{L}{m}$

Question121

A straight rod of length L extends from x = a to x = L + a. The gravitational force it exerts on point mass ‘m’ at x = 0,if the mass per unit length of the rod is $A + Bx^2$, is given by:
[12 Jan. 2019 I]

Options:

A. $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) - BL\right]$

B. $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) - BL\right]$

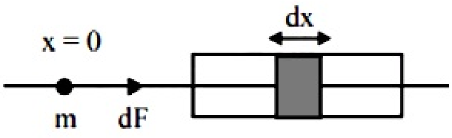
C. $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) + BL\right]$

D. $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

Answer: D

Solution:

Solution:
Given $\lambda = (A + Bx^2)$
Taking small element dm of length dx at a distance x from $x = 0$



so, $dm = \lambda dx$
$dm = (A + Bx^2)dx$
$dF = \frac{Gm\,dm}{x^2}$
$\Rightarrow F = \int_a^{a+L} \frac{Gm}{x^2}(A + Bx^2)dx$
$= Gm\left[-\frac{A}{x} + Bx\right]_a^{a+L}$
$= Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

Question122

The energy required to take a satellite to a height 'h' above Earth surface (radius of Eareth $= 6.4 \times 10^3 \text{km}$) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is:
[9 Jan. 2019 II]

Options:

A. $1.6 \times 10^3 \text{km}$

B. $3.2 \times 10^3 \text{km}$

C. $6.4 \times 10^3 \text{km}$

D. $28 \times 10^4 \text{km}$

Answer: B

Solution:

Solution:
K.E. of satellite is zero at earth surface and at height h from energy conservation

$U_{surface} + E_1 = U_h$

$-\frac{GM_e m}{R_e} + E_1 = -\frac{GM_e m}{(Re + h)}$

$\Rightarrow E_1 = GM_e m\left(\frac{1}{R_e} - \frac{1}{R_e + h}\right) \Rightarrow E_1 = \frac{GM_e m}{(R_e + h)} \times \frac{h}{R_e}$

Gravitational attraction

$F_G = ma_c = \frac{mv^2}{(R_e + h)} = \frac{GM_e m}{(R_e + h)^2}$

$mv^2 = \frac{GM_e m}{(R_e + h)}$

$E_2 = \frac{mv^2}{2} = \frac{GM_e m}{2(R_e + h)}$

$E_1 = E_2$

Clearly, $\frac{h}{R_e} = \frac{1}{2} \Rightarrow h = \frac{R_e}{2} = 3200 \text{km}$

Question123

A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth collides with the satellite completely in elastically. The speeds of the satellite and the meteorite are the same, Just before the collision. The subsequent motion of the combined body will be
[12 Jan. 2019 I]

Options:

A. such that it escape to infinity

B. In an elliptical orbit

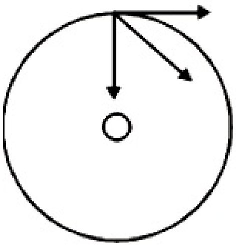
C. in the same circular orbit of radius R

D. in a circular orbit of a different radius

Answer: B

Solution:

Solution:



$m v \hat{i} + m v \hat{j} = 2m\vec{v}$

$\Rightarrow \vec{v} = \frac{v}{2}\hat{i} + \frac{v}{2}\hat{j}$

$\Rightarrow \vec{v} = \sqrt{\left(\frac{v}{2}\right)^2 + \left(\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}} \times \sqrt{\frac{GM}{R}}$

Question124

Two satellites, A and B, have masses m and 2m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, T_A / T_B, is :
[12 Jan. 2019 II]

Options:

A. $\frac{1}{2}$

B. 1

C. 2

D. $\sqrt{\frac{1}{2}}$

Answer: B

Solution:

Solution:

Orbital, velocity, $v = \sqrt{\frac{GM}{r}}$

Kinetic energy of satellite A,

$T_A = \frac{1}{2} m_A V_A^2$

Kinetic energy of satellite B,

$T_B = \frac{1}{2} m_B V_B^{\,2}$

$\Rightarrow \frac{T_A}{T_B} = \frac{m \times \frac{GM}{R}}{2m \times \frac{GM}{2R}} = 1$

Question125

A satellite is revolving in a circular orbit at a height h from the earth surface, such that h < < R where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is:
[11 Jan. 2019 I]

Options:

A. $\sqrt{2gR}$

B. $\sqrt{gR}$

C. $\sqrt{\frac{gR}{2}}$

D. $\sqrt{gR}(\sqrt{2} - 1)$

Answer: D

Solution:

Solution:
For a satellite orbiting close to the earth, orbital velocity is given by
$v_0 = \sqrt{g(R+h)} \approx \sqrt{gR}$
Escape velocity (v_e) is
$v_e = \sqrt{2g(R+h)} \approx \sqrt{2gR}\ [\because h << R]$
$\Delta v = v_e - v_0 = (\sqrt{2} - 1)\sqrt{gR}$

Question126

A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is:
[10 Jan. 2019 I]

Options:

A. $2mv^2$

B. mv^2

C. $\frac{1}{2}mv^2$

D. $\frac{3}{2}mv^2$

Answer: B

Solution:

Solution:
At height r from center of earth, orbital velocity
$v = \sqrt{\frac{GM}{r}}$
By principle of energy conservation
KE of $^{c}m' + \left(-\frac{GMm}{r}\right) = 0 + 0$
($\because$ At infinity ,$PE = KE = 0$)
or KE of $'m' = \frac{GMm}{r} = \left(\sqrt{\frac{GM}{r}}\right)^2 m = mv^2$

Question127

Two stars of masses 3×10^{31}kg each, and at distance 2×10^{11}m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is:
(Take Gravitational constant $G = 66 \times 10^{-11} N m^2 kg^{-2}$)
[10 Jan. 2019 II]

Options:

A. $2.4 \times 10^4 m/s$

B. $1.4 \times 10^5 m/s$

C. $3.8 \times 10^4 m/s$

D. $2.8 \times 10^5 m/s$

Answer: D

Solution:

Solution:
Let M is mass of star m is mass of meteroite
By energy convervation between 0 and ∞.
$-\frac{GMm}{r} + \frac{-GMm}{r} + \frac{1}{2}mV_{ese}^{2} = 0 + 0$
$\therefore v = \sqrt{\frac{4GM}{r}} = \sqrt{\frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{10^{11}}}$
$\simeq 2.8 \times 10^5 m/s$

Question128

The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$ th of that of the Earth. If ' R ' is the radius of the Earth, what is the radius of the planet? (Take the planets to have the same mass density).
[12 April 2019 II]

Options:

A. $\frac{R}{3}$

B. $\frac{R}{4}$

C. $\frac{R}{9}$

D. $\frac{R}{2}$

Answer: D

Solution:

Solution:

$\frac{W_e}{W_p} = \frac{mg_e}{mg_p} = \frac{9}{4}$ or $\frac{g_e}{g_p} = \frac{9}{4}$

or $\frac{GM / R^2}{G(M / 9) / R_p^2} = \frac{9}{4}$

$\therefore R_p = \frac{R}{2}$

Question129

The value of acceleration due to gravity at Earth's surface is $9.8ms^{-2}$. The altitude above its surface at which the acceleration due to gravity decreases to $4.9ms^{-2}$, is close to : (Radius of earth $= 6.4 \times 10^6 m$)
[10 April 2019 I]

Options:

A. $2.6 \times 10^6 m$

B. $6.4 \times 10^6 m$

C. $9.0 \times 10^6 m$

D. $1.6 \times 10^6 m$

Answer: A

Solution:

Given
Acceleration due to gravity at a height h from earth's surface is
$g_h = g(1 + hR_e)^{-2} \Rightarrow 4.9 = 9.8\left(1 + \frac{h}{R_e}\right)^{-2}$
$\frac{1}{\sqrt{2}} = \left(1 - \frac{h}{R_e}\right)$ [as $h << R_e$]
$h = R_e(\sqrt{2} - 1)$
$h = 6400 \times 0.414\text{km} = 2.6 \times 10^6\text{m}$

Question130

A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M . The gravitational field at distance ' 3a ' from the centre will be:
[9 April 2019 I]

Options:

A. $\frac{2GM}{9a^2}$

B. $\frac{GM}{9a^2}$

C. $\frac{GM}{3a^2}$

D. $\frac{2GM}{3a^2}$

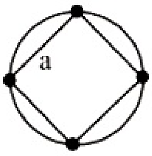
Answer: C

Solution:

Solution:
$E_g = \frac{GM}{(3a)^2} + \frac{G(2M)}{(3a)^2} = \frac{GM}{3a^2}$

Question131

Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of others' gravitational field in a circular orbit circumscribing the square ?



[8 April 2019 I]

Options:

A. $1.35\sqrt{\frac{GM}{a}}$

B. $1.16\sqrt{\frac{GM}{a}}$

C. $1.21\sqrt{\frac{GM}{a}}$

D. $1.41\sqrt{\frac{GM}{a}}$

Answer: B

Solution:

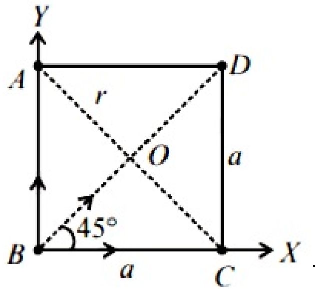
Solution:

$AC = a\sqrt{2} \quad \because r = \frac{AC}{2} = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$

Resultant force on the body

$$B = \frac{GM^2}{a^2}\hat{i} + \frac{GM^2}{a^2}\hat{j} + \frac{GM^2}{(a\sqrt{2})^2}\left(\cos 45^\circ\,\hat{i} + \sin 45^\circ\,\hat{j}\right)$$

$$\Rightarrow |F| = \frac{GM^2}{a^2}(\sqrt{2}) + \frac{GM^2}{2a^2}$$



$\frac{Mv^2}{r} =$ Resultant force towards centre

$$\therefore \frac{Mv^2}{\left(\frac{a}{\sqrt{2}}\right)} = \frac{GM^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$\Rightarrow v^2 = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow v = \sqrt{\frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right)} = 1.16\sqrt{\frac{GM}{a}}$$

Question132

A test particle is moving in circular orbit in the gravitational field produced by a mass density r(r) = $\frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T:
[8 April 2019 II]

Options:

A. T / R is a constant

B. T^2 / R^3 is a constant

C. T / R^2 is a constant

D. TR is a constant

Answer: A

Solution:

Solution:

$F = \frac{GMm}{r} = \int a \frac{\rho(dV)m}{r^2}$

$= mG \int_0^R \frac{k}{r^2} \frac{4\pi r^2 dr}{r^2}$

$= -4\pi kGm \left(\frac{1}{r}\right)_0^R$

$= -\frac{4\pi kGm}{R}$

Using Newton's second law, we have

$\frac{mv_0^{\ 2}}{R} = \frac{4\pi kGm}{R}$

or $v_0 = C$ (const.)

Time period, $T = \frac{2\pi R}{v_0} = \frac{2\pi R}{C}$

or $= \frac{T}{R} =$ constant.

Question133

A spaceship orbits around a planet at a height of 20km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet ? [Given : Mass of Planet $= 8 \times 10^{22}$kg, Radius of planet $= 2 \times 10^6$m, Gravitational constant $G = 6.67 \times 10^{-11} N\, m^2 / kg^2$]
[10 April 2019 II]

Options:

A. 9

B. 17

C. 13

D. 11

Answer: D

Solution:

Solution:

Time period of revolution of satellite,

$T = \frac{2\pi r}{v}$

$v = \sqrt{\frac{GM}{r}}$

$\therefore T = 2\pi r \sqrt{\frac{r}{GM}} = 2\pi \sqrt{\frac{r^3}{GM}}$

Substituting the values, we get

$$T = 2\pi\sqrt{\frac{(202)^3 \times 10^{12}}{6.67 \times 10^{-11} \times 8 \times 10^{22}}}\ \text{sec}$$

$T = 7812.2s$

$T \simeq 2.17hr \Rightarrow 11$ revolutions

Question134

A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon.
[8 April 2019 II]

Options:

A. $\frac{E}{64}$

B. $\frac{E}{32}$

C. $\frac{E}{4}$

D. $\frac{E}{16}$

Answer: D

Solution:

Solution:

Escape velocity,

$$v_c = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\rho V}{R}}$$

$$= \sqrt{\frac{2GS \times 4\pi R^3}{R}} = \sqrt{\frac{8}{3}\pi\rho GR^2}$$

For moon, $v_c' = \sqrt{\frac{8}{3}\pi\rho GR_m{}^2}$

Given, $\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi R_m{}^3$ or $R_m = \frac{R}{4}$

$$\therefore v_e' = \sqrt{\frac{8}{3}\pi\rho G\left(\frac{R}{4}\right)^2} = \frac{v_c}{4}$$

$$\frac{E}{E'} = \frac{\frac{1}{2}mv_e{}^2}{\frac{1}{2}mv'_e{}^2} = \frac{v_e{}^2}{v'_c{}^2} = \frac{v_e}{\left(\frac{v_e}{4}\right)} = 16$$

or $E' = \frac{E}{16}$

Question135

Take the mean distance of the moon and the sun from the earth to be

0.4×10^6km and 150×10^6km respectively. Their masses are 8×10^{22}kg and 2×10^{30}kg respectively. The radius of the earth is 6400km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is:
[Online April 15, 2018]

Options:

A. 2

B. 6

C. 10^{-2}

D. 0.6

Answer: A

Solution:

Solution:
As we know, Gravitational force of attraction,

$$F = \frac{GMm}{R^2}$$

$$F_1 = \frac{GM_e m}{r_1^2} \text{ and } F_2 = \frac{GM_e M_s}{r_2^2}$$

$$\Delta F_1 = \frac{2GM_e m}{r_1^3}\Delta r_1 \text{ and } \Delta F_2 = \frac{GM_e M_s}{r_2^3}\Delta r_2$$

$$\frac{\Delta F_1}{\Delta F_2} = \frac{m\Delta r_1}{r_1^3}\frac{r_2^3}{M_s \Delta r_2} = \left(\frac{m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$$

Using $\Delta r_1 = \Delta r_2 = 2R_{earth}$; $m = 8 \times 10^{22}$kg
$M_s = 2 \times 10^{30}$kg
$r_1 = 0.4 \times 10^6$km and $r_2 = 150 \times 10^6$km

$$\frac{\Delta F_1}{\Delta F_2} = \left(\frac{8 \times 10^{22}}{2 \times 10^{30}}\right)\left(\frac{150 \times 10^6}{0.4 \times 10^6}\right)^3 \times 1 \cong 2$$

Question136

Suppose that the angular velocity of rotation of earth is increased. Then, as a consequence.
[Online April 16, 2018]

Options:

A. There will be no change in weight anywhere on the earth

B. Weight of the object, everywhere on the earth, wild decrease

C. Weight of the object, everywhere on the earth, will increase

D. Except at poles, weight of the object on the earth will decrease

Answer: D

Solution:

With rotation of earth or latitude, acceleration due to gravity vary as $g' = g - \omega^2 R\cos^2\phi$
Where ϕ is latitude, there will be no change in gravity at poles as $\phi = 90°$
At all other points as ω increases g' will decreases hence, weight, $W = mg$ decreases.

Question137

A body of mass m is moving in a circular orbit of radius R about a planet of mass M . At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$, and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is:
[Online April 15, 2018]

Options:

A. $-\frac{GMm}{2R}$

B. $+\frac{GMm}{6R}$

C. $-\frac{GMm}{6R}$

D. $\frac{GMm}{2R}$

Answer: C

Solution:

Solution:
Initial gravitational potential energy, $E_i = -\frac{GMm}{2R}$
Final gravitational potential energy,
$E_f = -\frac{GMm/2}{2\left(\frac{R}{2}\right)} - \frac{GMm/2}{2\left(\frac{3R}{2}\right)} = -\frac{GMm}{2R} - \frac{GMm}{6R}$
$= -\frac{4GMm}{6R} = -\frac{2GMm}{3R}$
therefore Difference between initial and final energy,$E_f - E_i = \frac{GMm}{R}\left(-\frac{2}{3} + \frac{1}{2}\right) = -\frac{GMm}{6R}$

Question138

If the Earth has no rotational motion, the weight of a person on the equator is W. Determine the speed with which the earth would have to rotate about its axis so that the person at the equator will weight $\frac{3}{4}$W .

Radius of the Earth is 6400km and $g = 10m / s^2$.
[Online April 8, 2017]

Options:

A. 1.1×10^{-3} rad / s

B. 0.83×10^{-3} rad / s

C. 0.63×10^{-3} rad / s

D. 0.28×10^{-3} rad / s

Answer: C

Solution:

Solution:

We know, $g' = g - \omega^2 R\cos^2\theta$

$\frac{3g}{4} = g - \omega^2 R$

Given, $g' = \frac{3}{4}g$

$\omega^2 R = \frac{g}{4}$

$\omega = \sqrt{\frac{g}{4R}} = \sqrt{\frac{10}{4 \times 6400 \times 10^3}}$

$= \frac{1}{2 \times 8 \times 100} = 0.6 \times 10^{-3}$ rad / s

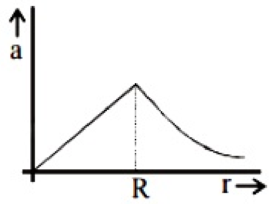
---

Question139

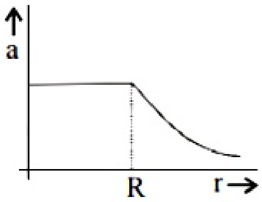
The mass density of a spherical body is given by $\rho(r) = \frac{k}{r}$ for $r \le R$ and $\rho(r) = 0$ for $r > R$
where r is the distance from the centre.
The correct graph that describes qualitatively the acceleration, a, of a test particle as a function of r is :
[Online April 9, 2017]

Options:

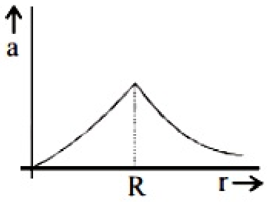
A.



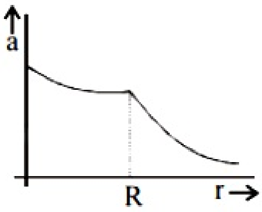
B.



C.



D.



Answer: B

Solution:

Solution:

Given that, mass density $\left(\frac{\text{mass}}{\text{volume}}\right)$ of a spherical body $\rho(r) = \frac{k}{r}$

$\frac{M}{V} = \frac{k}{r}$ for inside $r \leq R$

$M = \frac{kv}{r}$ ..... (i)

Inside the surface of sphere Intensity

$I = \frac{GM\,r}{R^3} \quad \because I = \frac{F}{m}$

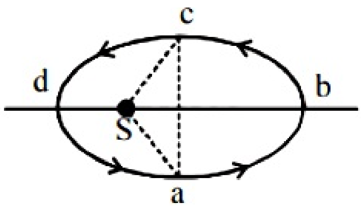
$g_{inside} = \frac{GM\,r}{R^3}$ or $I = \frac{mg}{m} = g$

$= \frac{G}{R^3} \cdot \frac{kv}{r} \cdot r =$ constant From eq. (i),

Similarly, $g_{out} = \frac{GM}{r^2}$

Question140

Figure shows elliptical path abed of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (See figure) With db as the semimajor axis, and ca as the semi minor axis. If t_1 is the time taken for planet to go over path abs and t_2 for path taken over cda then:



[Online April 9, 2016]

Options:

A. $t_1 = 4t_2$

B. $t_1 = 2t_2$

C. $t_1 = 3t_2$

D. $t_1 = t_2$

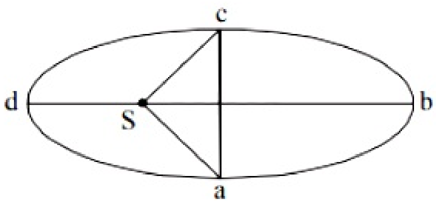
Answer: C

Solution:

Solution:
Let area of ellipse abcd =x

Area of SabcS $= \frac{x}{2} + \frac{x}{4}$ (i.e., ar of abca + SacS) (Area of half ellipse + Area of triangle)

$= \frac{3x}{4}$



Area of Sad cS $= x - \frac{3x}{4} = \frac{x}{4}$

$\frac{\text{Area ofSabcS}}{\text{Area of Sad cS}} = \frac{3x/4}{x/4} = \frac{t_1}{t_2}$

$\frac{t_1}{t_2} = 3$ or, $t_1 = 3t_2$

Question141

A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; h< < R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.)
[2016]

Options:

A. $\sqrt{gR/2}$

B. $\sqrt{gR}(\sqrt{2} - 1)$

C. $\sqrt{2gR}$

D. $\sqrt{gR}$

Answer: B

Solution:

Solution:
For $h << R$, the orbital velocity is $\sqrt{gR}$
Escape velocity $= \sqrt{2gR}$
$\therefore$ The minimum increase in its orbital velocity $= \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}(\sqrt{2} - 1)$

Question142

An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is :
[Online April 10, 2016]

Options:

A. Zero since astronaut feels weightless

B. $\frac{GMm}{(R+h)^2} < F_G < \frac{GMm}{R^2}$

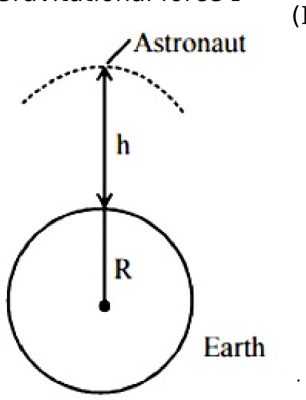
C. $F_G = \frac{GMm}{(R+h)^2}$

D. $0 < F_G < \frac{GMm}{R^2}$

Answer: C

Solution:

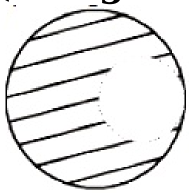
Solution:
According to universal law of Gravitation,
Gravitational force $F = \frac{GMm}{(R+h)^2}$



Question143

From a solid sphere of mass M and radius R, a spherical portion of

radius R/2 is removed, as shown in the figure.
Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is :
(G = gravitational constant)



[2015]

Options:

A. $\frac{-2GM}{3R}$

B. $\frac{-2GM}{R}$

C. $\frac{-GM}{2R}$

D. $\frac{-GM}{R}$

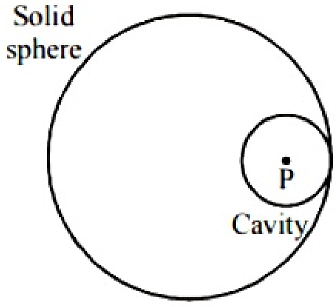
Answer: D

Solution:

Solution:
Due to complete solid sphere, potential at point P

$$V_{sphere} = \frac{-GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{-GM}{2R^3}\left(\frac{11R^2}{4}\right) = -11\frac{GM}{8R}$$



Due to cavity part potential at point P

$$V_{cavity} = -\frac{3}{2}\frac{\frac{GM}{8}}{\frac{R}{2}} = -\frac{3GM}{8R}$$

So potential at the centre of cavity

$$= V_{sphere} - V_{cavity}$$

$$= -\frac{11GM}{8R} - \left(-\frac{3}{8}\frac{GM}{R}\right) = \frac{-GM}{R}$$

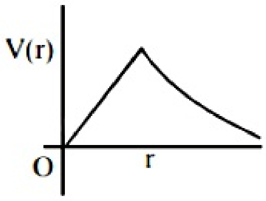
Question144

Which of the following most closely depicts the correct variation of the gravitational potential V(r) due to a large planet of radius R and

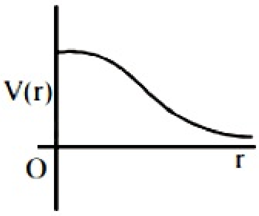
uniform mass density ? (figures are not drawn to scale) [Online April 11, 2015]

Options:

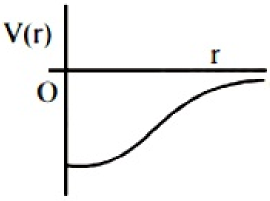
A.



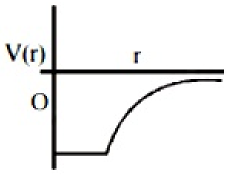
B.



C.



D.



Answer: C

Solution:

Solution:

As, $V = -\frac{GM}{2R^3}(3R^2 - r^2)$

Graph (c) most closely depicts the correct variation of v(r).

Question145

A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius R(R < < L). A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre.

If the time period of star is T and its distance from the galaxy's axis is r,

then :
[Online April 10, 2015]

Options:

A. $T \propto r$

B. $T \propto \sqrt{r}$

C. $T \propto r^2$

D. $T^2 \propto r^3$

Answer: A

Solution:

Solution:

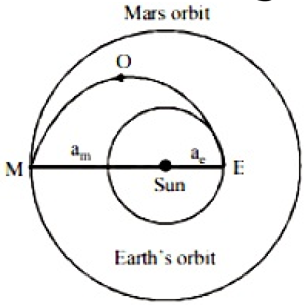
$F = \frac{2GM}{Lr} m$ or, $\frac{mv^2}{r} = \frac{2GM}{Lr} m$

$mr\left(\frac{2\pi}{T}\right)^2 = \frac{2GM\,m}{Lr}\left[\because v = r\omega \text{ and } \omega = \frac{2\pi}{T}\right]$

$\Rightarrow T \propto r$

Question146

India's Mangalyan was sent to the Mars by launching it into a transfer orbit EOM around the sun. It leaves the earth at E and meets Mars at M. If the semi-major axis of Earth's orbit is $a_e = 1.5 \times 10^{11}$m, that of Mars orbit $a_m = 2.28 \times 10^{11}$m, taken Kepler's laws give the estimate of time for Mangalyan to reach Mars from Earth to be close to:



[Online April 9, 2014]

Options:

A. 500 days

B. 320 days

C. 260 days

D. 220 days

Answer: B

Question147

Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is:
[2014]

Options:

A. $\sqrt{\frac{GM}{R}}$

B. $\sqrt{2\sqrt{2}\frac{GM}{R}}$

C. $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

D. $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

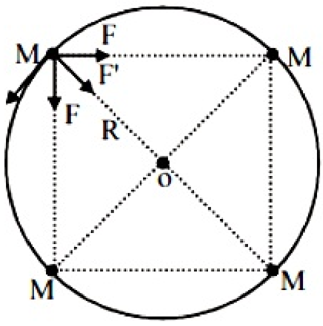
Answer: D

Solution:

Solution:

$2F\cos 45° + F' = \frac{Mv^2}{R}$ (From figure)

Where $F = \frac{GM^2}{(\sqrt{2}R)^2}$ and $F' = \frac{GM^2}{4R^2}$



$\Rightarrow \frac{2 \times GM^2}{\sqrt{2}(R\sqrt{2})^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$

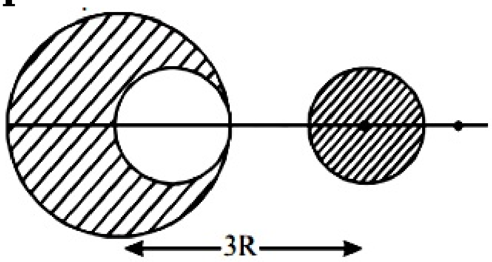
$\Rightarrow \frac{GM^2}{R}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right] = Mv^2$

$\therefore v = \sqrt{\frac{GM}{R}\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right)} = \frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

Question148

From a sphere of mass M and radius R, a smaller sphere of radius $\frac{R}{2}$ is carved out such that the cavity made in the original sphere is between

its centre and the periphery (See figure). For the configuration in the figure where the distance between the centre of the original sphere and the removed sphere is 3R, the gravitational force between the two sphere is:



[Online April 11, 2014]

Options:

A. $\frac{59GM^2}{450R^2}$

B. $\frac{41GM^2}{450R^2}$

C. $\frac{41GM^2}{3600R^2}$

D. $\frac{GM^2}{225R^2}$

Answer: A

Solution:

Solution:
Volume of removed sphere
$V_{remo} = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{4}{3}\pi R^3\left(\frac{1}{8}\right)$
Volume of the sphere (remaining)
$V_{remain} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi R^3\left(\frac{1}{8}\right) = \frac{4}{3}\pi R^3\left(\frac{7}{8}\right)$
Therefore mass of sphere carved and remaining sphere are at respectively $\frac{1}{8}M$ and $\frac{7}{8}M$.

Therefore, gravitational force between these two sphere, $F = \frac{GMm}{r^2} = \frac{G\frac{7M}{8} \times \frac{1}{8}M}{(3R)^2} = \frac{7}{64 \times 9}\frac{GM^2}{R^2}$

$\simeq \frac{41}{3600}\frac{GM^2}{R^2}$

Question149

The gravitational field in a region is given by $\vec{g} = 5N/kg\,\hat{i} + 12N/kg\,\hat{j}$. The change in the gravitational potential energy of a particle of mass 1kg when it is taken from the origin to a point (7m, −3m) is:
[Online April 19, 2014]

Options:

A. 71J

B. $13\sqrt{58}$J

C. -71J

D. 1J

Answer: D

Solution:

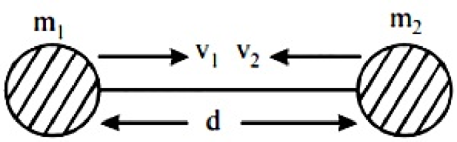
Gravitational field, $I = (5\hat{i} + 12\hat{j})N / kg$

$d r$

$$v = -\left[\int_0^x I_x dx + \int_0^y I_y dy\right]$$
$$= -[I_x . x + I_y . y]$$
$$= -[5(7-0) + 12(-3-0)]$$
$$= -[35 + (-36)] = 1J / kg$$

i.e., change in gravitational potential 1 J/kg.
Hence change in gravitational potential energy 1 J

Question150



Two hypothetical planets of masses m_1 and m_2 are at rest when they are infinite distance apart. Because of the gravitational force they move towards each other along the line joining their centres. What is their speed when their separation is 'd'?
(Speed of m_1 is v_1 and that of m_2 is v_2)
[Online April 12, 2014]

Options:

A. $v_1 = v_2$

B. $v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$ $v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

C. $v_1 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$ $v_2 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

D. $v_1 = m_2\sqrt{\frac{2G}{m_1}}$ $v_2 = m_2\sqrt{\frac{2G}{m_2}}$

Answer: B

Solution:

We choose reference point, infinity, where total energy of the system is zero.
So, initial energy of the system = 0

Final energy $= \frac{1}{2}m_1 v_1{}^2 + \frac{1}{2}m_2 v_2{}^2 - \frac{Gm_1 m_2}{d}$

From conservation of energy,
Initial energy = Final energy

$\therefore 0 = \frac{1}{2}m_1 v_1{}^2 + \frac{1}{2}m_2 v_2{}^2 - \frac{Gm_1 m_2}{d}$

or $\frac{1}{2}m_1 v_1{}^2 + \frac{1}{2}m_1 v_2{}^2 = \frac{Gm_1 m_2}{d}$ ...(1)

By conservation of linear momentum

$m_1 v_1 + m_2 v_2 = 0$ or $\frac{v_1}{v_2} = -\frac{m_2}{m_1} \Rightarrow v_2 = -m_1 m_2 v_1$

Putting value of v_2 in equation (1), we get

$m_1 v_1{}^2 + m_2 \left(-\frac{m_1 v_1}{m_2}\right)^2 = \frac{2Gm_1 m_2}{d}$

$\frac{m_1 m_2 v_1{}^2 + m_1{}^2 v_1{}^2}{m_2} = \frac{2Gm_1 m_2}{d}$

$v_1 = \sqrt{\frac{2Gm_2{}^2}{d\,(m_1 + m_2)}} = m_2 \sqrt{\frac{2G}{d\,(m_1 + m_2)}}$

Similarly $v_2 = -m_1 \sqrt{\frac{2G}{d\,(m_1 + m_2)}}$

Question151

The change in the value of acceleration of earth towards sun, when the moon comes from the position of solar eclipse to the position on the other side of earth in line with sun is:
(mass of the moon $= 7.36 \times 10^{22}$kg, radius of the moon's orbit $= 3.8 \times 10^8$m).
[Online April 22, 2013]

Options:

A. $6.73 \times 10^{-5} m / s^2$

B. $6.73 \times 10^{-3} m / s^2$

C. $6.73 \times 10^{-2} m / s^2$

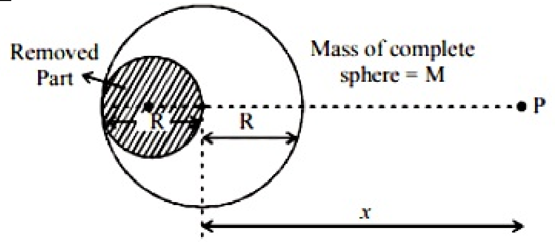
D. $6.73 \times 10^{-4} m / s^2$

Answer: A

Question152

The gravitational field, due to the 'left over part' of a uniform sphere (from which a part as shown, has been 'removed out'), at a very far off

point, P, located as shown, would be (nearly) :



[Online April 9, 2013]

Options:

A. $\frac{5}{6}\frac{GM}{x^2}$

B. $\frac{8}{9}\frac{GM}{x^2}$

C. $\frac{7}{8}\frac{GM}{x^2}$

D. $\frac{6}{7}\frac{GM}{x^2}$

Answer: C

Solution:

Solution:

Let mass of smaller sphere (which has to be removed) is m Radius $= \frac{R}{2}\left(\right.$ from figure)

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{m}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}$$

$$\Rightarrow m = \frac{M}{8}$$

Mass of the left over part of the sphere

$$M' = M - \frac{M}{8} = \frac{7}{8}M$$

Therefore gravitational field due to the left over part of the sphere $= \frac{GM'}{x^2} = \frac{7}{8}\frac{GM}{x^2}$

Question153

What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R?
[2013]

Options:

A. $\frac{5GmM}{6R}$

B. $\frac{2GmM}{3R}$

C. $\frac{GmM}{2R}$

D. $\frac{GmM}{3R}$

Answer: A

Solution:

Solution:
As we know,

Gravitational potential energy $= \frac{-GM\,m}{r}$

and orbital velocity, $v_0 = \sqrt{GM\ / R + h}$

$E_f = \frac{1}{2}mv_0^2 - \frac{GM\,m}{3R} = \frac{1}{2}m\frac{GM}{3R} - \frac{GM\,m}{3R}$

$= \frac{GM\,m}{3R}\left(\frac{1}{2} - 1\right) = \frac{-GM\,m}{6R}$

$E_i = \frac{-GM\,m}{R} + K$

$E_i = E_f$

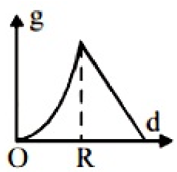
Therefore minimum required energy, $K = \frac{5GM\,m}{6R}$

Question154

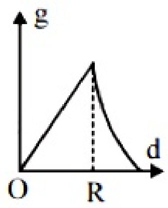
The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius): [Online May 7, 2012]

Options:

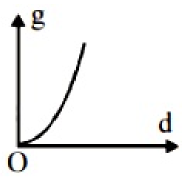
A.



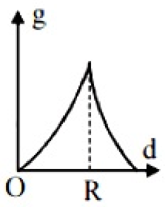
B.



C.



D.



Answer: B

Solution:

Solution:
Variation of acceleration due to gravity, g with distance 'd ' from centre of the earth

If $d < R$, $g = \frac{Gm}{R^2}d$ i.e., $g \propto d$ (straight line)

If $d = R$, $g_s = \frac{Gm}{R^2}$

If $d > R$, $g = \frac{Gm}{d^2}$ i.e., $g \propto \frac{1}{d^2}$

Question155

Assuming the earth to be a sphere of uniform density, the acceleration due to gravity inside the earth at a distance of r from the centre is proportional to
[Online May 12, 2012]

Options:

A. r

B. r^{-1}

C. r^2

D. r^{-2}

Answer: A

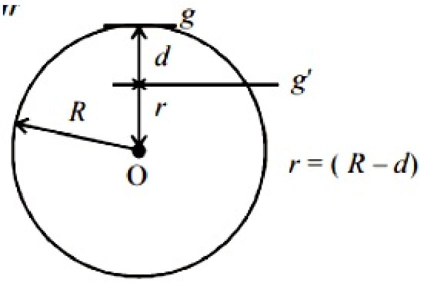
Solution:

Solution:
Acceleration due to gravity at depth d from the surface of the earth or at a distance r from the centre ‘O’ of the earth

$g' = \frac{4}{3}\pi\rho G r$

Hence $g' \propto r$

"



Question156

The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km respectively.
The required energy for this work will be
[2012]

Options:

A. 6.4×10^{11} Joules

B. 6.4×10^{8} Joules

C. 6.4×10^{9} Joules

D. 6.4×10^{10} Joules

Answer: D

Solution:

Solution:
The work done to launch the spaceship

$W = -\int_R^\infty \vec{F} \cdot d\vec{r} = -\int_R^\infty \frac{GMm}{r^2} dr$

$W = +\frac{GMm}{R}$ ..... (i)

The force of attraction of the earth on the spaceship, when it was on the earth's surface

$F = \frac{GMm}{R^2}$

$\Rightarrow mg = \frac{GMm}{R^2} \Rightarrow g = \frac{GM}{R^2}$ ..... (ii)

Substituting the value of g in (i) we get

$W = \frac{gR^2 m}{R}$

$\Rightarrow W = mgR$

$\Rightarrow W = 1000 \times 10 \times 6400 \times 10^3$

$= 6.4 \times 10^{10}$ Joule

Question157

A point particle is held on the axis of a ring of mass m and radius r at a distance r from its centre C. When released, it reaches C under the gravitational attraction of the ring. Its speed at C will be
[Online May 26, 2012]

Options:

A. $\sqrt{\frac{2Gm}{r}(\sqrt{2} - 1)}$

B. $\sqrt{\frac{Gm}{r}}$

C. $\sqrt{\frac{2Gm}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$

D. $\sqrt{\frac{2Gm}{r}}$

Answer: C

Solution:

Solution:
Let 'M' be the mass of the particle
Now, $E_{\text{initial}} = E_{\text{final}}$
i.e. $\frac{GMm}{\sqrt{2}r} + 0 = \frac{GMm}{r} + \frac{1}{2}MV^2$
or, $\frac{1}{2}MV^2 = \frac{GMm}{r}\left[1-\frac{1}{\sqrt{2}}\right]$
$\Rightarrow \frac{1}{2}V^2 = \frac{Gm}{r}\left[1-\frac{1}{\sqrt{2}}\right]$
or, $V = \sqrt{\frac{2Gm}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$

Question158

Two particles of equal mass ‘m’ go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is
[2011 RS]

Options:

A. $\sqrt{\frac{Gm}{4R}}$

B. $\sqrt{\frac{Gm}{3R}}$

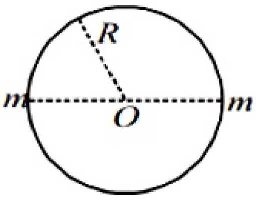
C. $\sqrt{\frac{Gm}{2R}}$

D. $\sqrt{\frac{Gm}{R}}$

Answer: A

Solution:

Solution:
As two masses revolve about the common centre of mass O.
∴ Mutual gravitational attraction = centripetal force



$$\frac{Gm^2}{(2R)^2} = m\omega^2 R$$

$$\Rightarrow \frac{Gm}{4R^3} = \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{Gm}{4R^3}}$$

If the velocity of the two particles with respect to the centre of gravity is v then

$v = \omega R$

$$v = \sqrt{\frac{Gm}{4R^3}} \times R = \sqrt{\frac{Gm}{4R}}$$

Question159

Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is: [2011]

Options:

A. $-\frac{4Gm}{r}$

B. $-\frac{6Gm}{r}$

C. $-\frac{9Gm}{r}$

D. zero

Answer: C

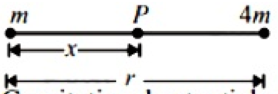
Solution:

Solution:

Let P be the point where gravitational field is zero.

$$\therefore \frac{Gm}{x^2} = \frac{4Gm}{(r-x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{2}{r-x} \Rightarrow r - x = 2x \Rightarrow x = \frac{r}{3}$$



Gravitational potential at P,

$$V = -\frac{Gm}{\frac{r}{3}} - \frac{4Gm}{\frac{2r}{3}} = -\frac{9Gm}{r}$$

Question160

The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is [2009]

Options:

A. $\frac{R}{\sqrt{2}}$

B. R / 2

C. $\sqrt{2}R$

D. 2R

Answer: D

Solution:

Solution:

On earth's surface $g = \frac{GM}{R^2}$

At height above earth's surface

$g_h = \frac{GM}{(R+h)^2}$

$\therefore \frac{g_n}{g} = \frac{R^2}{(R+h)^2}$

$\Rightarrow \frac{g/9}{g} = \left[\frac{R}{R+h}\right]^2$

$\Rightarrow \frac{R}{R+h} = \frac{1}{3}$

$\therefore h = 2R$

Question161

This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement-1 : For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides 4 π GM. and
Statement-2: If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.
[2008]

Options:

A. Statement -1 is false, Statement-2 is true

B. Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

C. Statement -1 is true, Statement-2 is true; Statement - 2 is not a correct explanation for Statement-1

D. Statement -1 is true, Statement-2 is false

Answer: B

Solution:

Solution:

Gravitational field, $E = -\frac{GM}{r^2}$

Flux, $\phi = \int \vec{E}_g . \vec{d}S = \left| E . 4\pi r^2 \right| = -4\pi GM$

where, M = mass enclosed in the closed surface

This relationship is valid when $\left|\vec{E}_g\right| \mu \frac{1}{r^2}$.

Question162

A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be
[2008]

Options:

A. 1.1 km s^{-1}

B. 11 km s^{-1}

C. 110 km s^{-1}

D. 0.11 km s^{-1}

Answer: C

Solution:

Solution:

Escape velocity on earth,

$$v_e = \sqrt{\frac{2GM_e}{R_e}} = 11 kms^{-1}$$

$$\therefore \frac{(v_e)_p}{(v_e)_e} = \frac{\sqrt{\frac{2GM_p}{R_p}}}{\sqrt{\frac{2GM_e}{R_e}}} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$$

$$= \sqrt{\frac{10M_e}{M_e} \times \frac{R_e}{R_e / 10}} = 10$$

$$\therefore (v_e)_p = 10 \times (v_e)_e = 10 \times 11 = 110 km / s$$

Question163

The change in the value of ' g ' at a height ' h ' above the surface of the earth is the same as at a depth ' d ' below the surface of earth. When both ' d ' and ' h ' are much smaller than the radius of earth, then which one of the following is correct?
[2005]

Options:

A. $d = \frac{3h}{2}$

B. $d = \frac{h}{2}$

C. $d = h$

D. $d = 2h$

Answer: D

Solution:

Solution:
Value of g with altitude is,
$g_h = g\left[1 - \frac{2h}{R}\right]$
Value of g at depth d below earth's surface,
$g_d = g\left[1 - \frac{d}{R}\right]$
Equating g_h and g_d, we get $d = 2h$

Question164

Average density of the earth
[2005]

Options:

A. is a complex function of g

B. does not depend on g

C. is inversely proportional to g

D. is directly proportional to g

Answer: D

Solution:

Solution:
Value of g on earth's surface,
$g = \frac{GM}{R^2} = \frac{G\rho \times V}{R^2}$

$\Rightarrow g = \dfrac{G \times \rho \times \frac{4}{3}\pi R^3}{R^2}$

$g = \frac{4}{3}\rho\pi G \,.\, R$ where $\rho \to$ average density

$\rho = \left(\dfrac{3g}{4\pi GR}\right)$

$\Rightarrow \rho$ is directly proportional to g.

Question165

A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere

(you may take $G = 6.67 \times 10^{-11} N\,m^2 / kg^2$)

[2005]

Options:

A. $3.33 \times 10^{-10} J$

B. $13.34 \times 10^{-10} J$

C. $6.67 \times 10^{-10} J$

D. $6.67 \times 10^{-9} J$

Answer: C

Solution:

Solution:

Initial P.E. $U_i = -\dfrac{GMm}{R}$

When the particle is far away from the sphere, the P.E. of the system is zero.

$\therefore U_f = 0$

$W = \Delta U = U_f - U_i = 0 - \left[\dfrac{-GMm}{R}\right]$

$W = \dfrac{6.67 \times 10^{-11} \times 100}{0.1} \times \dfrac{10}{1000} = 6.67 \times 10^{-10} J$

Question166

If ‘g’ is the acceleration due to gravity on the earth’s surface, the gain in the potential energy of an object of mass ‘m’ raised from the surface of the earth to a height equal to the radius ‘R' of the earth is

[2004]

Options:

A. $\frac{1}{4} mgR$

B. $\frac{1}{2}$mgR

C. 2mgR

D. mgR

Answer: B

Solution:

Solution:
On earth's surface potential energy,
$U = \frac{GmM}{R}$
At a height R from the earth's surface, P. E. of system $= -\frac{GmM}{2R}$
$\therefore \Delta U = \frac{-GmM}{2R} + \frac{GmM}{R}$
$\Rightarrow \Delta U = \frac{GmM}{2R}$
Now $\frac{GM}{R^2} = g; \therefore \frac{GM}{R} = gR$
$\therefore \Delta U = \frac{1}{2}mgR$

Question167

Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius 'R' around the sun will be proportional to [2004]

Options:

A. R^n

B. $R^{\left(\frac{n-1}{2}\right)}$

C. $R^{\left(\frac{n+1}{2}\right)}$

D. $R^{\left(\frac{n-2}{2}\right)}$

Answer: C

Solution:

Solution:
Gravitational force, $F = KR^{-n}$
This force provides the centripetal force $MR\omega^2$ to the planet at height h above earth's surface.
$\therefore F = KR^{-n} = MR\omega^2$
$\Rightarrow \omega^2 = KR^{-(n+1)}$
$\Rightarrow \omega = KR^{\frac{-(n+1)}{2}}$
$\frac{2\pi}{T} \propto R^{\frac{-(n+1)}{2}}$

$\therefore T \propto R^{\frac{+(n+1)}{2}}$

Question168

The time period of an earth satellite in circular orbit is independent of [2004]

Options:

A. both the mass and radius of the orbit

B. radius of its orbit

C. the mass of the satellite

D. neither the mass of the satellite nor the radius of its orbit

Answer: C

Solution:

Solution:
Time period of satellite is given by
$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$
Where R + h = radius of orbit of satellite
M = mass of earth.
Time period is independent of mass of satellite.

Question169

A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is [2004]

Options:

A. $\frac{gR^2}{R+x}$

B. $\frac{gR}{R-x}$

C. gx

D. $\left(\frac{gR^2}{R+x}\right)^{1/2}$

Answer: D

Solution:

Gravitational force provides the necessary centripetal force.
$\therefore$ Centripetal force on a satellite = Gravitational force

$$\therefore \frac{mv^2}{(R+x)} = \frac{GmM}{(R+x)^2} \text{ also } g = \frac{GM}{R^2}$$

$$\therefore \frac{mv^2}{(R+x)} = m\left(\frac{GM}{R^2}\right)\frac{R^2}{(R+x)^2}\frac{n!}{r!(n-r)!}$$

$$\therefore \frac{mv^2}{(R+x)} = mg\frac{R^2}{(R+x)^2}$$

$$\therefore v^2 = \frac{gR^2}{R+x} \Rightarrow v = \left(\frac{gR^2}{R+x}\right)^{1/2}$$

Question170

The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become [2003]

Options:

A. 10 hours

B. 80 hours

C. 40 hours

D. 20 hours

Answer: C

Solution:

Solution:
According to Kepler's law of periods $T^2 \propto R^3$

$$\therefore \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$$

$$\Rightarrow T_2 = T_1\left(\frac{R_2}{R_1}\right)^{\frac{3}{2}} = 5 \times \left[\frac{4R}{R}\right]^{3/2}$$

$= 5 \times 2^3 = 40$ hours

Question171

Two spherical bodies of mass M and 5M & radii R & 2R respectively are released in free space with initial separation between their centres equal to 12 R. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is [2003]

Options:

A. 2.5 R

B. 4.5 R

C. 7.5 R

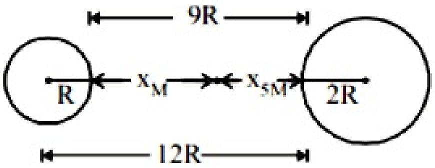
D. 1.5 R

Answer: C

Solution:

We know that

acceleration.



The gravitational force acting on both the masses is the same.

$F_1 = F_2$

$ma_1 = ma_2$

$\Rightarrow \frac{9M}{95M} = \frac{5M}{M} = 5$

$\Rightarrow \frac{9M}{95M} = \frac{1}{5}$ ....(i)

Let t be the time taken for the two masses to collide and x_{5M}, x_M be the distance travelled by the mass $5M$ and M respectively.

For mass 5M

$u = 0,$

$S = ut + \frac{1}{2}at^2$

$\therefore x_{5M} = \frac{1}{2}a_{5M}t^2$ ....(ii)

For mass M

$u = 0, s = x_M, t = t, a = a_M$

$\therefore s = ut + \frac{1}{2}at^2$

$\Rightarrow x_M = \frac{1}{2}a_M t^2$ ... (iii)

Dividing (ii) by (iii)

$\frac{x_{5M}}{x_M} = \frac{\frac{1}{2}a_{5M}t^2}{\frac{1}{2}a_M t^2} = \frac{a_{5M}}{a_M} = \frac{1}{5}$ [From (i)]

$\therefore 5x_{5M} = x_M$ ....(iv)

From the figure it is clear that

$x_{5M} + x_M = 9R$ ....(v)

Where O is the point where the two spheres collide.

From (iv) and (v)

$\frac{x_M}{5} + x_M = 9R$

$\therefore 6x_M = 45R$

$\therefore x_M = \frac{45}{6}R = 7.5R$

Question172

The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be
[2003]

Options:

A. $11\sqrt{2}$ km/s

B. 22 km/s

C. 11 km/s

D. $\frac{11}{\sqrt{2}}$ km/s

Answer: C

Solution:

Solution:
$v_e = \sqrt{2gR}$
Clearly escape velocity does not depend on the angle at which the body is projected.

Question173

Energy required to move a body of mass m from an orbit of radius 2R to 3R is
[2002]

Options:

A. $GM\,m / 12R^2$

B. $GM\,m / 3R^2$

C. $GM\,m / 8R$

D. $GM\,m / 6R$.

Answer: D

Solution:

Solution:
Gravitational potential energy of mass m in an orbit of radius R
$u = -\frac{GM\,m}{R}$
Energy required = potential energy at 3R - potential energy a 2R
$= \frac{-GM\,m}{3R} - \left(\frac{-GM\,m}{2R}\right)$
$= \frac{-GM\,m}{3R} + \frac{GM\,m}{2R}$
$= \frac{-2GM\,m + 3GM\,m}{6R} = \frac{GM\,m}{6R}$

Question174

The kinetic energy needed to project a body of mass m from the earth

surface (radius R) to infinity is
[2002]

Options:

A. mgR/2

B. 2mgR

C. mgR

D. mgR/4.

Answer: C

Solution:

Solution:
$K.E = \frac{1}{2}mv_e^2$
Here $v_e =$ escape velocity is independent of mass of the body
Escape velocity, $v_e = \sqrt{2gR}$
Substituting value of v_e in above equation we get
$K.E = \frac{1}{2}m \times 2gR = mgR$

Question175

If suddenly the gravitational force of attraction between Earth and a satellite revolving around it becomes zero, then the satellite will [2002]

Options:

A. continue to move in its orbit with same velocity

B. move tangentially to the original orbit in the same velocity

C. become stationary in its orbit

D. move towards the earth

Answer: B

Solution:

Solution:
Due to inertia of motion it will move tangentially to the original orbit with the same velocity.

Question176

The escape velocity of a body depends upon mass as [2002]

Options:

A. m^0

B. m^1

C. m^2

D. m^3

Answer: A

Solution:

Escape velocity, $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

$\Rightarrow v_e \propto m^0$

Where M , R are the mass and radius of the planet respectively. Clearly, escape velocity is independent of mass of the body

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